Demonstration of Tests of Conversion of PM Dot Notation to Parentheses

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SECTION 0. VERIFICATION TESTS (of dot to paren dot icn)
For each proposition is given:
1: the PM notation with dots.
2: the notation with parentheses
3: the Polish (with Lukasiewicz symbols) notation
4: the Polish notation from Dan O'Leary's translation for comparison

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According to PM to *56, page 9.

"The use of dots. Dots on the line of the symbols have two uses, one to bracket off propositions, the other to indicate the logical product of two propositions. Dots immediately preceded or followed by "$\vee$" or "$\supset$" or "$\equiv$" or "$\vdash$" or by "(x)", "$(x,y),''$ "(x,y,z)" ... or "$(\exists x),''$ "$(\exists x,y),''$ "$(\exists x,y,z)$"... or "$[(\imath x)(\phi x)]$" or "$[R`y]$" or analogous expressions , serve to bracket off a proposition ; dots occurring otherwise serve to mark a logical product. The general principle is that a larger number of dots indicates an outside bracket, a smaller number indicates an inside bracket. The exact rule as to the scope of the bracket indicated by dots is arrived at by dividing the occurrences of dots into three groups which we will name I, II, and III. Group I consists of dots adjoining a sign of implication ($\supset$) or of equivalence ($\equiv$) or of disjunction ($\vee$) or of equality by definition ($=$ Df). Group II consists of dots following brackets indicative of an apparent variable such as (x) or (x,y) or $(\exists x)$ or $(\exists x,y)$ or $[(\imath x)(\phi x)]$. Group III consists of dots which stand between propositions in order to indicate a logical product. The scope of the bracket indicated by any collection of dots extends backwards or forwards beyond any smaller number of dots, or any equal number from a group of less force, until we reach either the end of the asserted proposition or a greater number of dots or an equal number belonging to a group of equal or superior force. Dots indicating a logical product have a scope which works both backwards and forwards; other dots only work away from the adjacent sign of disjunction, implication, or equivalence, or forward from the adjacent symbol of one of the other kinds enumerated in Group II."

I intend to approach this as analogous to compiler design. I believe what I already did is OK so far as lexical analysis. I need to write a Chomsky grammar for PM (so far as concerns

dots).

Let A be dots of type I, B be dots of type II, and C be dots of type III.

First (without quantifiers)

Let P be a proposition, and O be any truth function other than conjunction.

T be a Term, and F be a factor.

P1 -> P2 A1 O A2 T1,

where the dots in A1 exceeds those in P2 and those in A2 exceed those in T1

P1 -> T1

T1 -> T2 C F1,

where the dots in C exceed those in T2 or F1

T1 -> F1

$*1{\cdot}01 \vdash .p. \supset .q. = .\sim p \vee q$

$*1{\cdot}01 \vdash (((p))) \supset (((q))) = (((\sim p \vee q)))$

$*1{\cdot}01 \vdash Cpq = ANpq$

*1.01 Cpq=ANpq

$*1{\cdot}2 \vdash :p \vee p. \supset .p$

$*1{\cdot}2 \vdash (((p \vee p))) \supset (((p)))$

$*1{\cdot}2 \vdash CAppp$

*1.2 CAppp Taut

$*1{\cdot}3 \vdash :q. \supset .p \vee q$

$*1{\cdot}3 \vdash (((q))) \supset (((p \vee q)))$

$*1{\cdot}3 \vdash CqApq$

*1.3 CqApq Add

$*1{\cdot}4 \vdash :p \vee q. \supset .q \vee p$

$*1{\cdot}4 \vdash (((p \vee q))) \supset (((q \vee p)))$

$*1{\cdot}4 \vdash CApqAqp$

*1.4 CApqAqp Perm

*1.5 in PM used parentheses ! I converted it to dot notation!

$*1{\cdot}5 \vdash :.p:\vee .q \vee r. \supset :q. \vee .p \vee r$

$*1{\cdot}5 \vdash ((((p)))) \vee (((q \vee r))) \supset ((((q)))) \vee (((p \vee r))))$

$*1{\cdot}5 \vdash CApAqrAqApr$

*1.5 CApAqrAqApr Assoc

$*1{\cdot}6 \vdash :.q \supset r. \supset :p \vee q. \supset .p \vee r$

$*1{\cdot}6 \vdash (((q \supset r))) \supset ((((p \vee q))) \supset (((p \vee r))))$

$*1{\cdot}6 \vdash CCqrCApqApr$

*1.6 CCqrCApqApr

$*2{\cdot}01 \vdash :p \supset \sim p. \supset .\sim p$

$*2{\cdot}01 \vdash (((p \supset \sim p))) \supset (((\sim p)))$

$*2{\cdot}01 \vdash CCpNpNp$

*2.01 CCpNpNp Abs

$*2{\cdot}02 \vdash :q. \supset .p \supset q$

$*2{\cdot}02 \vdash (((q))) \supset (((p \supset q)))$

$*2{\cdot}02 \vdash CqCpq$
*2.02 CqCpq Simp
$*2{\cdot}03 \vdash :p \supset \sim q . \supset . q \supset \sim p$
$*2{\cdot}03 \vdash (((p \supset \sim q))) \supset (((q \supset \sim p)))$
$*2{\cdot}03 \vdash CCpNqCqNp$
*2.03 CCpNqCqNp Transp
$*2{\cdot}04 \vdash :. p . \supset . q \supset r : \supset : q . \supset . p \supset r$
$*2{\cdot}04 \vdash ((((p)))) \supset (((q \supset r)))) \supset ((((q)))) \supset (((p \supset r))))$
$*2{\cdot}04 \vdash CCpCqrCqCpr$
*2.04 CCpCqrCqCpr Comm
$*2{\cdot}05 \vdash :. q \supset r . \supset : p \supset q . \supset . p \supset r$
$*2{\cdot}05 \vdash (((q \supset r))) \supset ((((p \supset q))) \supset (((p \supset r))))$
$*2{\cdot}05 \vdash CCqrCCpqCpr$
*2.05 CCqrCCpqCpr Syll
$*2{\cdot}06 \vdash :. p \supset q . \supset : q \supset r . \supset . p \supset r$
$*2{\cdot}06 \vdash (((p \supset q))) \supset ((((q \supset r))) \supset (((p \supset r))))$
$*2{\cdot}06 \vdash CCpqCCqrCpr$
*2.06 CCpqCCqrCpr Syll
$*2{\cdot}07 \vdash :p . \supset . p \vee p$
$*2{\cdot}07 \vdash (((p))) \supset (((p \vee p)))$
$*2{\cdot}07 \vdash CpApp$
*2.07 CpApp
$*2{\cdot}08 \vdash . p \supset p$
$*2{\cdot}08 \vdash (p \supset p)$
$*2{\cdot}08 \vdash Cpp$
*2.08 Cpp Id
$*2{\cdot}1 \vdash . \sim p \vee p$
$*2{\cdot}1 \vdash (\sim p \vee p)$
$*2{\cdot}1 \vdash ANpp$
*2.1 ANpp
$*2{\cdot}11 \vdash . p \vee \sim p$
$*2{\cdot}11 \vdash (p \vee \sim p)$
$*2{\cdot}11 \vdash ApNp$
*2.11 ApNp
*2.12, *2.13 and *2.14 also use parentheses in PM. I converted them.
$*2{\cdot}12 \vdash . p \supset \sim\sim p$
$*2{\cdot}12 \vdash (p \supset \sim\sim p)$
$*2{\cdot}12 \vdash CpNNp$
*2.12 CpNNp
$*2{\cdot}13 \vdash . p \vee \sim\sim\sim p$
$*2{\cdot}13 \vdash (p \vee \sim\sim\sim p)$
$*2{\cdot}13 \vdash ApNNNp$

*2.13 ApNNNp

$*2{\cdot}14 \vdash .\sim\sim p \supset p$

$*2{\cdot}14 \vdash (\sim\sim p \supset p)$

$*2{\cdot}14 \vdash CNNpp$

*2.14 CNNpp

$*2{\cdot}15 \vdash :\sim p \supset q \,.\supset.\, \sim q \supset p$

$*2{\cdot}15 \vdash (((\sim p \supset q))) \supset (((\sim q \supset p)))$

$*2{\cdot}15 \vdash CCNpqCNqp$

*2.15 CCNpqCNqp

$*2{\cdot}16 \vdash :p \supset q \,.\supset.\, \sim q \supset \sim p$

$*2{\cdot}16 \vdash (((p \supset q))) \supset (((\sim q \supset \sim p)))$

$*2{\cdot}16 \vdash CCpqCNqNp$

*2.16 CCpqCNqNp

$*3{\cdot}47 \vdash :. p \supset r \,.\, q \supset s \,.\supset: p \,.\, q \,.\supset.\, r \,.\, s$

$*3{\cdot}47 \vdash (((p \supset r) \land (q \supset s))) \supset ((((p) \land (q))) \supset (((r) \land (s))))$

$*3{\cdot}47 \vdash CKCprCqsCKpqKrs$

*3.47 CKCprCqsCKpqKrs

$*4{\cdot}22 \vdash :p \equiv q \,.\, q \equiv r \,.\supset.\, p \equiv r$

$*4{\cdot}22 \vdash (((p \equiv q) \land (q \equiv r))) \supset (((p \equiv r)))$

$*4{\cdot}22 \vdash CKEpqEqrEpr$

*4.22 CKEpqEqrEpr

$*4{\cdot}41 \vdash :. p \,.\lor.\, q \,.\, r : \equiv .\, p \lor q \,.\, p \lor r$

$*4{\cdot}41 \vdash ((((p))) \lor (((q) \land (r)))) \equiv (((p \lor q) \land (p \lor r)))$

$*4{\cdot}41 \vdash EApKqrKApqApr$

*4.41 EApKqrKApqApr

$*4{\cdot}43 \vdash :. p \,.\equiv: p \lor q \,.\, p \lor \sim q$

$*4{\cdot}43 \vdash (((p))) \equiv ((((p \lor q) \land (p \lor \sim q))))$

$*4{\cdot}43 \vdash EpKApqApNq$

*4.43 EpKApqApNq

$*4{\cdot}44 \vdash :. p \,.\equiv: p \,.\lor.\, p \,.\, q$

$*4{\cdot}44 \vdash (((p))) \equiv ((((p))) \lor (((p) \land (q))))$

$*4{\cdot}44 \vdash EpApKpq$

*4.44 EpApKpq

$*4{\cdot}87 \vdash :. p \,.\, q \,.\supset.\, r : \equiv : p \,.\supset.\, q \supset r : \equiv : q \,.\supset.\, p \supset r : \equiv : q \,.\, p \,.\supset.\, r$

$*4{\cdot}87 \vdash ((((p) \land (q))) \supset (((r)))) \equiv ((((p))) \supset (((q \supset r)))) \equiv ((((q))) \supset (((p \supset r)))) \equiv ((((q) \land (p))) \supset (((r))))$

$*4{\cdot}87 \vdash EEECKpqrCpCqrCqCprCKqpr$

*4.87 EEECKpqrCpCqrCqCprCKqpr

$*4{\cdot}88 \vdash :. p \,.\, q \,.\supset.\, r \,.\equiv: p \,.\supset.\, q \supset r : \equiv : q \,.\supset.\, p \supset r : \equiv : q \,.\, p \,.\supset.\, r$

$*4{\cdot}88 \vdash (((p) \land (q))) \supset (((r))) \equiv ((((p))) \supset (((q \supset r)))) \equiv ((((q))) \supset (((p \supset r)))) \equiv ((((q) \land (p))) \supset (((r))))$

$*4{\cdot}88 \vdash EEECKpqrCpCqrCqCprCKqpr$

Not in PM
$*5{\cdot}33 \vdash :. p \,.\, q \supset r \,.\equiv: p : p \,.\, q \,.\supset .\, r$
$*5{\cdot}33 \vdash (((p) \wedge (q \supset r))) \equiv ((((p)) \wedge ((p) \wedge (q))) \supset (((r))))$
$*5{\cdot}33 \vdash EKpCqrCKpKpqr$
*5.33 EKpCqrKpCKpqr
From Landon D. C. Elkind's Paper in Russell: Vol. 43, no. 1, page 44
$*431{\cdot}441 \vdash .p \vee q \,.\equiv .\, r \supset s$
$*431{\cdot}441 \vdash (((p \vee q))) \equiv (((r \supset s)))$
$*431{\cdot}441 \vdash EApqCrs$
$*431{\cdot}442 \vdash :p \,.\vee .\, q \equiv r : \supset : s$
$*431{\cdot}442 \vdash ((((p))) \vee (((q \equiv r)))) \supset ((((s))))$
$*431{\cdot}442 \vdash CApEqrs$
$*431{\cdot}443 \vdash :p \vee q \,.\equiv .\, r : \supset : s$
$*431{\cdot}443 \vdash ((((p \vee q))) \equiv (((r)))) \supset ((((s))))$
$*431{\cdot}443 \vdash CEApqrs$
$*431{\cdot}444 \vdash :p : \vee : q \equiv r \,.\supset .\, s$
$*431{\cdot}444 \vdash ((((p)))) \vee ((((q \equiv r))) \supset (((s))))$
$*431{\cdot}444 \vdash ApCEqrs$
$*431{\cdot}445 \vdash :p : \vee : q \,.\equiv .\, r \supset s$
$*431{\cdot}445 \vdash ((((p)))) \vee ((((q))) \equiv (((r \supset s))))$
$*431{\cdot}445 \vdash ApEqCrs$
From same, page 54
$*431{\cdot}54 \vdash :p \,.\, q \,.\, r \,.\, s : \supset : p \,.\, s : r \,.\, q$
$*431{\cdot}54 \vdash ((((p) \wedge (q) \wedge (r) \wedge (s)))) \supset ((((p) \wedge (s)) \wedge ((r) \wedge (q))))$
$*431{\cdot}54 \vdash CKKKpqrsKKpsKrq$