

# Demonstration of Tests of Conversion of PM Dot Notation to Parentheses

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January 31, 2024

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## SECTION 0. VERIFICATION TESTS (of dot to paren dot icn)

For each proposition is given:

- 1: the PM notation with dots.
  - 2: the notation with parentheses
  - 3: the Polish (with Lukasiewicz symbols) notation
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\*2•06†:  $p \supset q . \supset : q \supset r . \supset . p \supset r$

Version with parentheses

\*2•06†  $(p \supset q) \supset ((q \supset r) \supset (p \supset r))$

Polish Lukasiewicz notation

\*2•06†  $CCpqCCqrCpr$

\*3•47†:  $p \supset r . q \supset s . \supset : p . q . \supset . r . s$

Version with parentheses

\*3•47†  $(p \supset r) \wedge (q \supset s) \supset ((p) \wedge (q) \supset (r) \wedge (s))$

Polish Lukasiewicz notation

\*3•47†  $CKCprCqsKCKpqrs$

\*4•22†:  $p \equiv q . q \equiv r . \supset . p \equiv r$

Version with parentheses

\*4•22†  $(p \equiv q) \wedge (q \equiv r) \supset (p \equiv r)$

Polish Lukasiewicz notation

\*4•22†  $CKEpqEqrEpr$

\*4•41†:  $p . \vee . q . r : \equiv . p \vee q . p \vee r$

Version with parentheses

\*4•41†  $((p) \vee (q) \wedge (r)) \equiv (p \vee q) \wedge (p \vee r)$

Polish Lukasiewicz notation

\*4•41†  $KEKApqrApqApr$

\*4•43†:  $p . \equiv : p \vee q . p \vee \sim q$

Version with parentheses

\*4•43†  $(p) \equiv ((p \vee q) \wedge (p \vee \sim q))$

Polish Lukasiewicz notation

\*4•43†  $EpKApqApNq$

\*4·44†:  $p \equiv p \vee p \cdot q$

Version with parentheses

\*4·44†  $(p) \equiv ((p) \vee (p) \wedge (q))$

Polish Lukasiewicz notation

\*4·44†  $EpKAppq$

\*4·87†:  $p \cdot q \supset r \equiv p \supset q \supset r \equiv q \supset p \supset r \equiv q \cdot p \supset r$

Version with parentheses

\*4·87†  $((p) \wedge (q) \supset (r)) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$

Polish Lukasiewicz notation

\*4·87†  $EECKpqrCpCqrCqCprCKqpr$

\*4·88†:  $p \cdot q \supset r \equiv p \supset q \supset r \equiv q \supset p \supset r \equiv q \cdot p \supset r$

Version with parentheses

\*4·88†  $(p) \wedge (q) \supset (r) \equiv ((p) \supset (q \supset r)) \equiv ((q) \supset (p \supset r)) \equiv ((q) \wedge (p) \supset (r))$

Polish Lukasiewicz notation

\*4·88†  $EECKpqrCpCqrCqCprCKqpr$

\*5·33†:  $p \cdot q \supset r \equiv p : p \cdot q \supset r$

Version with parentheses

\*5·33†  $(p) \wedge (q \supset r) \equiv (p) \wedge ((p) \wedge (q) \supset (r))$

Polish Lukasiewicz notation

\*5·33†  $KEKpCqrpCKpqr$

From Landon D. C. Elkind's Paper in Russell: Vol. 43, no. 1, page 44

\*431·441†:  $p \vee q \equiv r \supset s$

Version with parentheses

\*431·441†  $(p \vee q) \equiv (r \supset s)$

Polish Lukasiewicz notation

\*431·441†  $EApqCrs$

\*431·442†:  $p \vee q \equiv r : \supset : s$

Version with parentheses

\*431·442†  $((p) \vee (q \equiv r)) \supset ((s))$

Polish Lukasiewicz notation

\*431·442†  $CApEqrs$

\*431·443†:  $p \vee q \equiv r : \supset : s$

Version with parentheses

\*431·443†  $((p \vee q) \equiv (r)) \supset ((s))$

Polish Lukasiewicz notation

\*431·443†  $CEApqrs$

\*431·444†:  $p : \vee : q \equiv r \cdot \supset \cdot s$

Version with parentheses

\*431·444†  $(p) \vee ((q \equiv r) \supset (s))$

Polish Lukasiewicz notation

\*431·444†  $ApCEqrs$

\*431·445†:  $p : \vee : q \equiv r \supset s$

Version with parentheses

\*431·445†  $(p) \vee ((q) \equiv (r \supset s))$

Polish Lukasiewicz notation

\*431·445†  $ApEqCrS$

From same, page 54

\*431·54†  $:p.q:r.s:\supset:p.s:r.q$

Version with parentheses

\*431·54†  $((p) \wedge (q)) \wedge ((r) \wedge (s)) \supset ((p) \wedge (s)) \wedge ((r) \wedge (q))$

Polish Lukasiewicz notation

\*431·54†  $KCKKpqKrsKpsKrq$

check longer prop name

Version with parentheses

Polish Lukasiewicz notation

Propositions involving quantifiers

\*9·2†  $(x) . \psi x . \supset . \psi y$

Version with parentheses

\*9·2†  $((x))\psi x \supset (\psi y)$

Polish Lukasiewicz notation

\*9·2†  $C(x)\psi x\psi y$

\*9·21†  $:(x) . \psi x \supset \phi x . \supset : (x) . \psi x . \supset . (x) . \phi x$

Version with parentheses

\*9·21†  $((x))\psi x \supset \phi x \supset (((x))\psi x \supset ((x))\phi x)$

Polish Lukasiewicz notation

\*9·21†  $CCC(x)\psi x\phi x(x)\psi x(x)\phi x$

\*9·22†  $:(x) . \psi x \supset \phi x . \supset : (\mathbf{E}x) . \psi x . \supset . (\mathbf{E}x) . \phi x$

Version with parentheses

\*9·22†  $((x))\psi x \supset \phi x \supset (((\mathbf{E}x))\psi x \supset ((\mathbf{E}x))\phi x)$

Polish Lukasiewicz notation

\*9·22†  $CCC(x)\psi x\phi x(\mathbf{E}x)\psi x(\mathbf{E}x)\phi x$

\*9·31†  $:(\mathbf{E}x) . \phi x . \vee . (\mathbf{E}x) . \phi x : \supset . (\mathbf{E}x) . \phi x$

Version with parentheses

\*9·31†  $((((\mathbf{E}x))\phi x) \vee ((\mathbf{E}x))\phi x) \supset ((\mathbf{E}x))\phi x$

Polish Lukasiewicz notation

\*9·31†  $CA(\mathbf{E}x)\phi x(\mathbf{E}x)\phi x(\mathbf{E}x)\phi x$

\*9·401†  $::p:\vee:q.\vee.(\mathbf{E}x).\psi x::\supset::q:\vee:p.\vee.(\mathbf{E}x).\psi x$

Version with parentheses

\*9·401†  $((p) \vee ((q) \vee ((\mathbf{E}x))\psi x)) \supset ((q) \vee ((p) \vee ((\mathbf{E}x))\psi x))$

Polish Lukasiewicz notation

\*9·401†  $C ApAq(\mathbf{E}x)\psi xAqAp(\mathbf{E}x)\psi x$

\*10·35†  $:(\mathbf{E}x) . p . \psi x . \equiv : p : (\mathbf{E}x) . \psi x$

Version with parentheses

\*10·35†  $((\mathbf{E}x))p \wedge (\psi x) \equiv (p) \wedge (((\mathbf{E}x))\psi x)$

Polish Lukasiewicz notation

\*10·35†  $KEK(\mathbf{E}x)p\psi xp(\mathbf{E}x)\psi x$

\*11·2†  $(x, y) . \phi[x, y] . \equiv . (y, x) . \phi[x, y]$

Version with parentheses

$$*11\cdot2\vdash ((x, y) \cdot \phi[x, y]) \equiv ((y, x) \cdot \phi[x, y])$$

Polish Lukasiewicz notation

$$*11\cdot2\vdash E(x, y) \cdot \phi[x, y](y, x) \cdot \phi[x, y]$$

One Step in proof of 11.55 I wanted example of 2 adjacent quantifiers - hard to find.

$$*11\cdot551\vdash \vdash (x) \vdash (\forall y) \cdot \psi x \cdot \phi[x, y] \cdot \equiv \vdash \psi x \vdash (\forall y) \cdot \phi[x, y]$$

Version with parentheses

$$*11\cdot551\vdash (((x))(\forall y)\psi x) \wedge (\phi[x, y]) \equiv (\psi x) \wedge (((\forall y))\phi[x, y])$$

Polish Lukasiewicz notation

$$*11\cdot551\vdash KEK(x)(\forall y)\psi x\phi[x, y]\psi x(\forall y)\phi[x, y]$$

From same, page 46

$$*431\cdot46\vdash (x) \cdot \psi x \cdot \phi x \cdot \supset \cdot (x) \cdot \psi x$$

Version with parentheses

$$*431\cdot46\vdash (((x))\psi x) \wedge (\phi x) \supset ((x))\psi x$$

Polish Lukasiewicz notation

$$*431\cdot46\vdash CK(x)\psi x\phi x(x)\psi x$$

Other Tests

$$*99\cdot99\vdash \vdash (\exists x) \vdash \sim \psi x \cdot \supset \cdot (x) \cdot \sim \psi x$$

Version with parentheses

$$*99\cdot99\vdash ((\sim(\exists x))\sim \psi x) \supset ((x))\sim \psi x$$

Polish Lukasiewicz notation

$$*99\cdot99\vdash CN(\exists x)N\psi x(x)N\psi x$$