§1 Notation

The letters 'a', 'b', and 'c' with or without any primes will be used as the abbreviations names of simples. Elementary proposition will be written as a capital letter followed by any number of names of simples, separated by commas and enclosed by parenthesis, eg. R(a, b).

The following things will be assumed:

1 All elementary propositions are propositions.

2 If $(\bar{p})$ is any group of propositions '$N(\bar{p})$' is a proposition which is true if all the propositions in the group $(\bar{p})$ are false and '$N(\bar{p})$' is false otherwise.

The letters 'p', 'q', and 'r' will be used as unspecified propositions. I now define

D1 *1.01 $\ulcorner p \mid q \urcorner \stackrel{df}{=} \ulcorner N(p,q) \urcorner$

D2 *1.02 $\ulcorner \sim p \urcorner \stackrel{df}{=} \ulcorner p \mid p \urcorner$

D3 $\ulcorner p . q \urcorner \stackrel{df}{=} \ulcorner \sim p \mid \sim q \urcorner$

D4 $\ulcorner p \vee q \urcorner \stackrel{df}{=} \ulcorner \sim (p \mid q) \urcorner$

D5 $\ulcorner p \supset q \urcorner \stackrel{df}{=} \ulcorner \sim p \vee q \urcorner$

D6 $\ulcorner p \equiv q \urcorner \stackrel{df}{=} \ulcorner (p \supset q) . (q \supset p) \urcorner$

I am using quasi quotation as defined by Quine in <u>Mathematical Logic</u>.

D7 Now let $(\bar{p})$ be all the propositions which can be obtained by putting in an appropriate name of some simple in for the name of a simple in a proposition. I define

$$\ulcorner (\exists \alpha) \phi \urcorner \stackrel{df}{=} \ulcorner N(N(\bar{p})) \urcorner$$

In this, ϕ is the name of an arbitrary

sign with is the result of putting
α for the name of a simple in
a proposition.

D8 $*1.08$ $\ulcorner(\alpha)\urcorner \overset{df}{=} \ulcorner\sim(\exists\alpha)\sim\urcorner$

D9 Let $(\bar{p})$ be all the propositions which
can be obtained by putting in
the names of any two appropriate
simples in the place of two names
of simples in a proposition.

$\ulcorner(\exists\beta)(\exists\alpha)\phi\urcorner \overset{df}{=} \ulcorner N(N(\bar{p})\urcorner$, etc

From now on, the Greek letters
ϕ, ψ, and χ will be used
for expressions which will
become propositions if a number
of variables ($\alpha, \beta, \gamma \cdot$) are
replaced by the names of
appropriate simples. The occurrence
of any number of existential
quantifiers is defined by D7 and
D9. The occurrence of universal
quantifiers is defined by D8.

The closure of 'ϕ' is the proposition which results from applying in alphabetic order ($x, y, z, x', y', z', \ldots$) the corresponding universal quantifier for every variable in the expression. That the closure of 'ϕ' is a theorem will be written $\vdash \ulcorner \phi \urcorner$.

§2 Primitive Propositions.

I have so far only made definitions. Before anything can be proven, something must be assumed. The following are descriptions of all the propositions I will assume to be logically true. They come from _Mathematical Logic_ by Quine. I have modified the notation some, and the generation of all propositions from elementary propositions has been made explicit.

*100 If ϕ is tautologous, by a truth table test, $\vdash \phi$.

*101 $\vdash \ulcorner (\alpha)(\phi \supset \psi) \supset ((\alpha)\phi \supset (\alpha)\psi) \urcorner$

*102 If ϕ' is like ϕ except for containing free occurrences of α' wherever ϕ contains free occurrences of α, then $\vdash \ulcorner (\alpha)\phi \supset \phi' \urcorner$

*104 If $\vdash \ulcorner p \supset q \urcorner$ and $\vdash p$, then $\vdash q$.

~~7.5 If $\ulcorner N(\bar{p}) \urcorner$ and~~

~~7.4 If $\vdash \ulcorner p \supset N(\bar{p}) \urcorner$ and $\vdash p$, and q is in $(\bar{p})$, then $\vdash \ulcorner \sim q \urcorner$~~

*105 $\vdash \ulcorner N(\bar{p}) \supset \sim p \urcorner$ where p is in $(\bar{p})$.

*106 If for any p in $(\bar{p})$, $\vdash \sim p$, then
$\vdash \ulcorner N(\bar{p}) \urcorner$

*107 If for any p in $(\bar{p})$ there is a q in $(\bar{q})$ such that $\vdash \ulcorner q \supset p \urcorner$
then $\vdash \ulcorner N(\bar{p}) \supset N(\bar{q}) \urcorner$

*108 $\vdash \ulcorner N(\bar{p}) . N(\bar{q}) \supset N(\bar{r}) \urcorner$
where any r in $(\bar{r})$ is also in $(\bar{p})$ or $(\bar{q})$.

§7 Identity & Quantification of functions

Let $(\bar{p})$ be the class of all elementary propositions of the form $\ulcorner \sim(\phi \equiv \psi) \urcorner$ where ϕ is like ψ except for containing a where ψ contains b, then I define:

D10 $\ulcorner a = b \urcorner \overset{df}{=} \ulcorner N(\bar{p}) \urcorner$

Let $(\bar{p})$ be the class of all propositions ψ which contain a proposition in a number of places (≥ 0) [illegible]
~~the same argument the~~

D11 $\ulcorner (\exists \phi)\, \psi \urcorner \overset{df}{=} N(N(\bar{p}))$

D12 $\ulcorner (\phi) \urcorner \overset{df}{=} \ulcorner \sim(\exists \phi) \sim \urcorner$

*180 If ϕ and ψ are the same except that ϕ contains a where ψ contains b, ~~then~~ then,

$$\vdash \ulcorner a = b . \supset . \phi \equiv \psi \urcorner$$

Proof: $\vdash \ulcorner a = b \supset N(\bar{p}) \urcorner$ *100 + D 10

$\vdash \ulcorner N(\bar{p}) \supset \sim(\sim(\phi \equiv \psi)) \urcorner$ D10 + *105

$\vdash \ulcorner \sim(\sim(\phi \equiv \psi)) \supset \phi \equiv \psi \urcorner$ *100

$\vdash \ulcorner a = b \supset \phi \equiv \psi \urcorner$ *112

~~*181 If ϕ and ψ are alike except that ϕ contains α where ψ contains β then~~

~~$\vdash \ulcorner (\beta)(\beta = \alpha . \supset \psi) \equiv \phi \urcorner$~~

~~$\vdash$~~

*181 $\vdash \ulcorner a = a \urcorner$

Let (p) be the set of all elem props which contain a. For any prop (p), $\sim(p \equiv p)$ is contradictory, so

$\vdash \ulcorner N(\bar{p}) \urcorner$ *106

$\vdash \ulcorner N(\bar{p}) \supset . a = a \urcorner$ D 10

$\vdash \ulcorner a = a \urcorner$ *112

$*182 \vdash \ulcorner a = b \supset b = a \urcorner$

$\vdash \ulcorner a = b . \supset N(\bar{p}) \urcorner$ D10

Let $(\bar{q})$ be the set of all propositions of the form $\ulcorner \sim(\psi \equiv \phi) \urcorner$ ϕ & ψ as in D10. Then for any p in $(\bar{p})$ there is a q in $(\bar{q})$ such that $q \supset p$.

$\vdash \ulcorner N(\bar{p}) \supset N(\bar{q}) \urcorner$ $*107$

$\vdash \ulcorner N(\bar{q}) \supset . b = a \urcorner$ D10 $*100$

$\vdash \ulcorner a = b . \supset . b = a \urcorner$ $*112$

$*183 \vdash \ulcorner b = a \supset a = b \urcorner$

$*182$

$*184 \vdash \ulcorner a = b \equiv b = a \urcorner$

$\vdash \ulcorner [182] \supset . [183] \supset [182] . [183] \urcorner$ $*100$

$\vdash \ulcorner [182] \urcorner$ $*182$

$\vdash \ulcorner [183] \supset [182] . [183] \urcorner$ $*104$

$\vdash \ulcorner 183 \urcorner$ $*183$

$\vdash [182] . [183]$ $*104$

[illegible]

$\vdash \ulcorner [182] . [183] \supset [184] \urcorner$ D6 + *100.
$\vdash \ulcorner [184] \urcorner$ *104

*185 $\vdash \ulcorner a = b . b = c . \supset . a = c \urcorner$

$\vdash \ulcorner a = b . b = c . \supset . N(\bar{p}) . N(\bar{q}) \urcorner$ *100 + D6.
$\vdash \ulcorner N(\bar{p}) . N(\bar{q}) . \supset N(\bar{r}) \urcorner$
where $(\bar{r})$ is the [illegible] of all
[illegible] propositions [illegible] . . .

$\vdash \ulcorner N(\bar{r}) \supset . a = c \urcorner$ *100 + D6.

$\vdash \ulcorner a = b . b = c . \supset . a = c \urcorner$ *112.

*186 $\vdash \ulcorner (x)(x = x) \urcorner$

let $(\bar{p})$ be all [illegible] of the form $\ulcorner \sim (a = a) \urcorner$
therefore $(\bar{p})$ [illegible] (p) [illegible] (*151)
$\vdash \ulcorner N(\bar{p}) \urcorner$ *106
$\vdash \ulcorner N(\bar{p}) \supset \sim (\exists x) \sim (x = x) \urcorner$ *100 + D7
$\vdash \ulcorner \sim (\exists x) \sim (x = x) \supset (x)(x = x) \urcorner$ *100 + D8
$\vdash \ulcorner N(\bar{p}) \supset (x)(x = x) \urcorner$ *112
$\vdash \ulcorner (x)(x = x) \urcorner$ *104

D1 $\ulcorner \phi \mid \psi \urcorner \overset{df}{=} \ulcorner N(\phi, \psi) \urcorner$

D2 $\ulcorner \sim \phi \urcorner \overset{df}{=} \ulcorner N(\phi) \urcorner$

D3 $\ulcorner (\phi . \psi) \urcorner \overset{df}{=} \ulcorner (\sim \phi \mid \sim \psi) \urcorner$

D4 $\ulcorner (\phi \vee \psi) \urcorner \overset{df}{=} \ulcorner \sim (\phi \mid \psi) \urcorner$

D5 $\ulcorner (\phi \supset \psi) \urcorner \overset{df}{=} \ulcorner (\sim \phi \vee \psi) \urcorner$

D6 $\ulcorner (\phi \equiv \psi) \urcorner \overset{df}{=} \ulcorner (\phi \supset \psi) . (\psi \supset \psi) \urcorner$

D7 $\ulcorner (\exists \alpha) \phi \urcorner \overset{df}{=} \ulcorner \sim (N(\bar{p})) \urcorner$, where $(\bar{p})$ is all the propositions which can be obtained by putting in the name of some appropriate simple in for the name of a simple in a proposition and ϕ is that proposition with α substituted for the name of the simple.

D8 $\ulcorner (\alpha) \urcorner \overset{df}{=} \ulcorner \sim (\exists \alpha) \sim \urcorner$

∗100 If ϕ is tautologous, $\vdash \phi$.

∗105 If for any ϕ in $(\bar{p})$ there is a ψ in $(\bar{q})$ such that $\ulcorner \psi \supset \phi \urcorner$, then $\vdash \ulcorner N(\bar{p}) \supset N(\bar{q}) \urcorner$

∗106 $\vdash \ulcorner (\alpha)(\phi \supset \psi) \supset . (\alpha)\phi \supset (\alpha)\psi \urcorner$

Proof

$\vdash \ulcorner (\alpha)(\phi \supset \psi) \supset \sim(\exists\alpha)\sim(\phi \supset \psi) \urcorner$ ∗100 + D8

$\vdash \ulcorner \sim(\exists\alpha)\sim(\phi \supset \psi) \supset N(\bar{p}) \urcorner$ ∗100 + D7

where $(\bar{p})$ is all the propositions obtained from $\sim(\phi \supset \psi)$ by substituting the names of an appropriate [illegible] for α

$\vdash \ulcorner N(\bar{p}) \supset N(\bar{q}) \urcorner$ ∗105

where