A proposition is anything that is true or false. In mathematics we are concerned with propositions which are true or false solely because of their logical structure. Propositions which are true solely because of their logical structure are called analytic.

In creating our logical [illegible], we must specify 3 groups of things: our logical [illegible], our primitive propositions, and our rules of inference.

But before we introduce these, it is necessary to introduce some notation for describing our [illegible]. This notation is not a [illegible] part of our system but is used to talk about it.

Greek letters [illegible] 'ε', 'ι', 'φ', and 'λ' will be used [illegible] of unspecified [illegible].

[illegible] letters will be used [illegible] of unspecified entities ([illegible]).

Quotation marks will be used to refer to the name of an expression.
Eg. "Europe" contains six letters.

__Corners__ are used to refer to the specific content of unspecified expressions.

Eg., If we use ϕ for 'Jones is away' and ψ for 'Smith is ill', then $\ulcorner \phi \equiv \psi \urcorner$ is 'Jones is away $\equiv$ Smith is ill.'

A __propositional function__ is produced by replacing an entity in a proposition by a variable, e.g. if 'the apple' in 'the apple is red' is replaced by x, the propositional function $\ulcorner$x is red' is produced.

We are now prepared to introduce our 3 primitives.

The first, $\ulcorner(\phi \downarrow \psi)\urcorner$ is a method of statement composition, i.e. creating a new proposition from two old ones. $\ulcorner(\phi \downarrow \psi)\urcorner$ is the __joint denial__ of ϕ and ψ, and is true only in the case both ϕ and ψ are false.

$\ulcorner(\phi \downarrow \psi)\urcorner$

ϕ	$\downarrow$	ψ
T	F	T
T	F	F
F	F	T
F	T	F

The second primative ⌜(x)⌝ is a method of turning a propositional function into a proposition. It is the *universal quantifier* and is used to assert a propositional function for all the values of its variable.

Eg ⌜(x)(x is red)⌝ asserts everything is red.

The third primative ⌜(α ∈ β)⌝ means almost the same thing as it usually does in mathematics. In the case where β is a class, it means α *is a member of* β; if β is not a class, it means α *equals* β.

We are now prepared to introduce some more notation necessary for describing our primative propositions.

Tautology:
To determine whether a statement is *tautologous*, we only need to see if it is true for all possible truth values of the atomic propositions compounded to form it.

Eg

$$\ulcorner((\emptyset \downarrow (\emptyset \downarrow \emptyset)) \downarrow ((\emptyset \downarrow \emptyset) \downarrow \emptyset))\urcorner$$

T F T F T   T   T F T F T
F F F T F   T   F T F F F

The Closure of a formula (proposition or propositional function) is obtained by applying whatever quantifiers are necessary (in alphabetic order) to turn the formula into a proposition. The closure of a proposition is the proposition.

Eg. The closure of the prop function $(x < y \equiv . y > x)$ is $(y)(x)(x < y \equiv . y > x)$ and is true

The closure of the prop function $(z)(x > z)$ is $(x)(z)(x > z)$ and is false.

Stratification In my last speech I mentioned Russell's contradiction. Some sets are not members of themselves e.g. the set of all dogs is not a dog; and some sets are members of themselves e.g. the set of all not-dogs. Now consider the set of all sets that are not members of themselves. If it is a member of itself then it cannot be a member of itself; and if it is not a member of itself it must be a member of itself.

We need a way in which we can, in general, eliminate all the classes which will lead us to such troubles. Stratification is such a method. We will call a formula *stratified* if it is possible to put numerals from the variables (the same numeral for all occurrences of the same variable) in such a way that the '∈' comes to be flanked always by consecutive ascending numerals.

Eg.

$$(x)((x \in y) \downarrow ((z)((y \in z) \downarrow (x \in y))))$$

is stratified since we can put

$$(4)((3 \in 4) \downarrow ((5)((4 \in 5) \downarrow (3 \in 4))))$$

The occurrence of a variable α is *free* iff α is not *bound* to a quantifier.

Some handy abbreviations

statement composition

$\ulcorner \sim \phi \urcorner \ \underline{\underline{df}}\ \ulcorner (\phi \downarrow \phi) \urcorner$

$\ulcorner (\phi . \psi) \urcorner \ \underline{\underline{df}}\ \ulcorner (\sim \phi \downarrow \sim \psi) \urcorner$

$\ulcorner (\phi \vee \psi) \urcorner \ \underline{\underline{df}}\ \ulcorner \sim (\phi \downarrow \psi) \urcorner$

$\ulcorner (\phi \supset \psi) \urcorner \ \underline{\underline{df}}\ \ulcorner (\sim \phi \vee \psi) \urcorner$

$\ulcorner (\phi \equiv \psi) \urcorner \ \underline{\underline{df}}\ \ulcorner ((\phi \supset \psi) . (\psi \supset \phi)) \urcorner$

existential quantification

$\ulcorner (\exists \alpha) \urcorner \ \underline{\underline{df}}\ \ulcorner \sim (\alpha) \sim \urcorner$

misc concerning membership

$\ulcorner (\zeta \tilde{\in} \eta) \urcorner \ \underline{\underline{df}}\ \ulcorner \sim (\zeta \in \eta) \urcorner$

$\ulcorner (\zeta_1, \zeta_2, \ldots, \zeta_n \in \eta) \urcorner \ \underline{\underline{df}}\ \ulcorner (\zeta_1 \in \eta . \zeta_2 \in \eta . \ldots . \zeta_n \in \eta) \urcorner$

identity

$\ulcorner (\zeta = \eta) \urcorner \ \underline{\underline{df}}\ \ulcorner (\alpha)(\alpha \in \zeta . \equiv . \alpha \in \eta) \urcorner$

$\ulcorner (\zeta \neq \eta) \urcorner \ \underline{\underline{df}}\ \ulcorner \sim (\zeta = \eta) \urcorner$

abstraction

$\ulcorner (\beta \in \hat{\alpha} \phi) \urcorner \ \underline{\underline{df}}\ \ulcorner (\exists \gamma)(\beta \in \gamma . (\alpha)(\alpha \in \gamma . \supset \phi)) \urcorner$

$\ulcorner (\hat{\alpha} \phi \in \zeta) \urcorner \ \underline{\underline{df}}\ \ulcorner (\exists \beta)(\beta = \hat{\alpha} \phi . \beta \in \zeta) \urcorner$

universal & null classes

'V' $\underline{\underline{df}}$ '$\hat{x}(x = x)$'

'Λ' $\underline{\underline{df}}$ '$\hat{x}(x \neq x)$'

axiom

We will use the assertion sign, $\vdash$, to mean that the closure of the formula following it is an axiom or theorem, depending on the situation.

We will use an infinite number of axioms. They are described as follows.

*100 If ϕ is tautologous, $\vdash \phi$

*101 $\vdash \ulcorner(\alpha)(\phi \supset \psi) \supset . (\alpha)\phi \supset (\alpha)\psi\urcorner$

*102 If α is not free in ϕ, $\vdash \ulcorner \phi \supset (\alpha)\phi \urcorner$

*103 If ϕ' is like ϕ except for containing free occurrences of α' wherever ϕ contains free occurrences of α, then $\vdash \ulcorner(\alpha)\phi \supset \phi'\urcorner$

*200 If ϕ has no free variables beyond $\alpha, \beta_1, \beta_2, \ldots, \beta_n$, and is formed from a stratified formula by restricting all bound variables to elements,
$\vdash \ulcorner \beta_1, \beta_2, \ldots, \beta_n \in V . \supset . \hat{\alpha}\phi \in V\urcorner$

*201 If ϕ is atomic, and ϕ' is formed from ϕ by putting α' for an occurrence of α, then $\vdash \ulcorner \alpha = \alpha' . \supset . \phi \supset \phi'\urcorner$

*202 If β is not α nor free in ϕ,

$$\vdash \ulcorner(\exists\beta)(\alpha)(\alpha \in \beta . \equiv . \alpha \in V . \phi)\urcorner$$

Rule of inference – modus ponens

*104 If ⌜$\phi \supset \psi$⌝ and ϕ are theorems, so is ψ.