The History of Formal Logic from Boole to today

by

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version of speech
to Math Club
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De Morgan (1806-1871)

In logical tradition. Worked out algebra of logic inferior to Boole's. Founder of logic of relations.

Boole (1815-1864)

Broke from logical tradition. Wants logic to be just like algebra. Primarily concerned with the legal manipulation of symbols, not their interpretation. Does believe these laws represent the fundamental laws of the operation of the mind.

The operations in Boole's system

Suppose x and y are arbitrary sets
The sum of x and y, $x + y$, is the union of x and y. (they must have no elements in common)
(We would write $(x \cup y)$.).
The product of x and y, $x \cdot y$, is the intersection of x and y. (We would write $x \cap y$)
The difference of x and y, $x - y$, is the set which contains the portion of x which is not in y.
(y must be a subset of x). (We would write $x \cap \bar{y}$).

1 = universal set
0 = null set

Boole's formal principles

(1) $x \cdot y = y \cdot x$;
(2) $x + y = y + x$;
(3) $z \cdot (x+y) = z \cdot x + z \cdot y$;
(4) $z \cdot (x-y) = z \cdot x - z \cdot y$;
(5) If $x = y$, then (a) $z \cdot x = z \cdot y$,
(b) $z + x = z + y$,
and (c) $x - z = y - z$;
(6) $x^2 = x$.

One of Boole's proofs

He assumed he could use any result of calculus.

Maclaurin expansion

$$\phi(x) = \phi(0) + \phi'(0) \cdot x + \phi''(0) \cdot \frac{x^2}{1 \cdot 2} + \cdots$$

using (c)

$$\phi(x) = \phi(0) + x \cdot [\phi'(0) + \phi''(0)/1 \cdot 2 + \cdots]$$

Sub 1 for x

$$\phi(1) = \phi(0) + [\cdots]$$

$$[\cdots] = \phi(1) - \phi(0)$$

so

$$\phi(x) = \phi(0) + x \cdot [\phi(1) - \phi(0)]$$

correct result

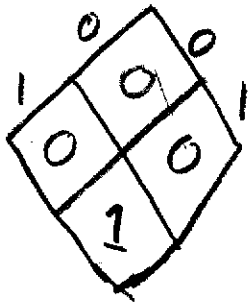
Jevons (1835-1882) } Improved Boolels
Schröder (1841-1902) } system

Peirce (1839-1914) founder of semiotics

Pragmaticism — The "real" meaning of any concept, doctrine, proposition, word, or any other signs is obtained by condidering what practical consequences necessary follow from the truth of the concept. Believes in scientific method. There is a real world independent of our opinions, to which truth refers.

Not to be confused with pragmatism (James) — A statement is true if believing it has results we like.

Peirces truth table
A and B



0 for false
1 for true

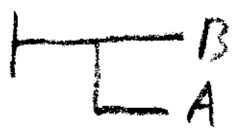
Frege (1848-1925)

Considered now by many as the greatest of the 19th century logicians. Less concerned about analogy between algebra & symbolic logic. Created a powerful logical system, but used very cumbersome notation. Defined cardinal numbers in logical terms and logically deduced arithmetic.

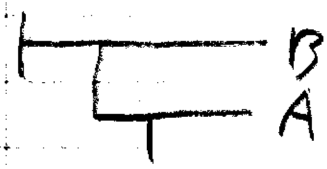
Examples of notation

$\vdash A$ It is judged that A is true.


" " " " Not-A is true.


" " " " A implies B is true,


" " " " A and B is true

 Acknowledged that A or B is true.

 " " " " for all a, F(a) is true

 " " " " for some a, F(a) is true

Frege was believed in <u>logicism</u>, ie, he believed he could define all the undefined concepts in arithmetic and prove all the unproven [illegible] therein by reducing them into logical terms.

Peano (1858 - 1932)

Logic weaker than Frege's logic, but still makes improvements. First to distinguish between class membership (∈) and class inclusion (⊂). Also uses a much more convienient notation than Frege. Wants to cover all of mathematics but is satisfied to deduce the various branches of mathematics from suitable axiom systems.

Peano's Axioms for Natural numbers

(1) 0 is a natural number
(2) Every natural number has a subsequent
(3) No natural number has 0 as a subsequent.
(4) Two different numbers have different subsequents.
(5) If some property T holds for zero, and if whenever T holds for any number x, it also holds for the subsequent of x, then T holds for every number n.

From these axioms, the entire theory of natural numbers can be constructed. They show a strong influence of Dedekind on Peano.

Whitehead (1861-1947)
Russell (1872-1970)
Combine the rigorous logic of Frege with the more convenient notation of Peano to create
PRINCIPIA MATHMATICA

Simplifications
Wiener

Proof Theory & completeness
Gödel
Hilbert
Tarski

Other solutions to Russell paradox
Zermelo
Neumann-Bernays
Quine

~~Logical Positivism~~
Wittgenstein (1889-1951)
Carnap (1891-
Einstein