An Introduction

To the Philosophy of

Logical Atomism

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1 Atomic Facts and the World

The ideas of logical simple or object and of structure seem to me to be indefinable. I will, consequently, not define them. Their meaning should become less vague as the various topics, with which they are concerned, are covered. An atomic fact is a structure of objects.[1] All of the atomic facts are independent of each other, i.e. the existence or non-existence of one cannot affect the existence or non-existence of another.[2] The world is the totality of atomic facts.[3]

What atomic facts are possible depends upon what objects there happen to be. A particular object cannot have any structure with other objects. An object may appear in structures which others cannot appear in. The form of an object is the possibility of its occurence in atomic facts.[4]

2 Elementary Propositions

Gottlob Frege made one of the main advances of modern logic when he replaced the notion of subject and predicate with those of (propositional) function and argument.[5] This opened the way to Wittgenstein theory of elementary propostions. An elementary propostion asserts an atomic fact. It does this by showing a relationship between the names of objects. This relation ship corresponds to the structure of the objects making up the atomic fact. Let a and b be objects, i.e. let "a" and "b" be the names of simples. Then in the elementary proposition "R (a,b)."[6] "a" and "b" are the arguments of the elementary propostion and the way they are related in it is the function they appear in to say "R (a,b)". Generally, an atomic fact might be a structure of any cardinal number of objects; that is there is no logical restriction on the number of objects in any atomic fact, though there might actually be a maximum in the world as it happens to be.

Now, can there be elementary propostion for any atomic fact? At any rate, an elementary propostion may assert any atomic fact, which is a structure of a finite cardinal number greater than one of objects. So an elemenatry proposition may have any cardinal number greater than one of arguments. If an elementary proposition had only one argument, it could not establish a relationship between its aruuments, so it would not, after all, be an elementary proposition. Letting a, b, c, and d be simples, "R (a,b)" would be an elementary porposition with two arguments; "S (a,b,c)" would be an elementary proposition with three arguments; and "T (a,b,c,d)" would be an elementary proposition with four arguments. It should not be important to logic how large the number of arguments appearing in an elementary proposition may be, this being an empirical question.[7] However the following diffuculty arises (or so it seems to me). How can there be elementary propositions to assert atomic facts which are structures of an infinite cardinal numer of objects? If the number is countable, i.e. can be set in one-to-one correspondence with the natural numbers, then the elementary propostion might be "R (a_0, a_1, a_2, a_3, . . .)" where there is a general way of knowing what a_n is if n is a natural number. However, if the number of objects in an atomic fact is uncountable, I can see no obvious way and elementary proposition could be (there is a way) made to assert the atomic fact. Wittgenstein does mention atomic facts which are infinitely complex,[8] but perhaps this is another of his deliberate absurdities. That there may be atomic facts that there is no to assert, in no way affects the theory that whatever elementary propositions there are, they do indeed assert atomic ~~fac~~ facts.

3 Propositions

In most logical systems, propositions, in addition to the elementary propositions, are generated in two ways : truth composition and quantification. I will first discuss truth composition, as the theory is set up by Henry Maurice Sheffer.[9] He used, in addition to the primitive idea of being an elementary proposition, the primitive idea of being a proposition, and that of a binary truth combination | on propositions. If "p" and "q" are propositions, "p|q" is true if both "p" and "q" are false and "p|q" is false if either (or both) are ture. The following properties of proposition are then given.

1. All elementary propositions are propositions
2. Whenever "p" and "q" are propositions, "p|q" is a proposition.

   The definition $\sim p \overset{df}{=} p|p$ is made of negation.
3. When ever p and the indicated combinations of p are propostions,,

   $\sim(\sim p) = p$.
4. Whenever p, q, and the indicated combinations of p and q are propostions,

   $p|(q|\sim q) = \sim p$.
5. Whenever p, q, r and the indicated combinations of p, q, and r are propositions $\sim(p|(q|r)) = (\sim q|p)|(\sim r|p)$.

The definition of conjunction, disjunction, implication, and equivolence can be made in terms of Sheffer's primitive as follows:

$p.q \overset{df}{=} \sim p|\sim q$

$p \vee q \overset{df}{=} \sim(p|q)$

$p \supset q \overset{df}{=} \sim p \vee q$

$p \equiv q \overset{df}{=} (p \supset q).(q \supset p).$ [10]

Sheffer shows that his five properties are consistent and independent. They are also as complete as the way truth composition is developed in Principia Mathematica. Note that the symbols "$\overset{df}{=}$", "≡" and " = " all have different uses. "$\overset{df}{=}$" is used to indicate that what appears on the left is an abbreviation for what appears on the right. "=" is used to indicate that what appears on the left is the same proposition as what appears on the right. " $\equiv$ " is a truth composition of the propostiion which is only true if both are true or both are false.

Now, observe that, since this is only a binary operation, the number of elementary propositions occuring in any proposition so generated will have to be some natural number and hence finite. This is why in systems using binary truth operations, quantifications must be added, in order to get propositions which contain the notions of all or some. Thus let a and b be simples and x and y be variables, whose values are simple. Then if "R (a,b)" is an elementary proposition "(x,y) R (x,y)" is the proposition that says whenever the names of two appropriate simples are substituted for "x" and "y" in "R(x,y)" a true elementary proposition results. Note, however, if the same name say "a" is substituted for both "x" and "y", the result is nonsense. As "R(a,a)" would assert that the same simple could occur at two different places in a structure, which is absurd, I will not discuss quantification further, because Wittgenstein defines it in terms of his form of truth composition.

4 Wittgenstein's General Form of Propositions

Russell's explanation[11] of Wittgenstein's notation $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ seems to me to be wrong. Wittgenstein defines $[a, x, o'x]$ to be the general term of the series of forms a, o'a, o'o'a,....[12]. So $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ should be the general term of the series of forms $\bar{p}$, N($\bar{p}$), N (N($\bar{p}$))..., leaving as yet undetermined what ($\bar{p}$) and N are. Now ($\bar{p}$) must be same set of proposition. N($\bar{p}$) is the negation of all the propositions in ($\bar{p}$).[13] Suppose Wittgenstein means by ($\bar{p}$) the set of all elementary propostions, and that ($\bar{p}$) =(p,q), then $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ would be the general term of the series of forms. (p.q), (~p.~q), (pvq), (~p.~q)..... but this does not contain

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such forms as "$p \supset q$". So Wittgenstein would be wrong, if this were what he meant. I think, however that Wittgenstein did not mean this. I think the following may be a correct interpretation of Wittgenstein theory.

1 All elementary propositions are propositions.

2 A class of propostions can be stipulated by giving a description of all the propositions in the class. T his will be written $(\bar{p})$ where the propositions in $(\bar{p})$ are not all necessarially elementary propositions.

3 If $(\bar{p})$ is such a class of propositions, $N(\bar{p})$ is a proposition. $N(\bar{q})$ is true if all the propositions in $(\bar{p})$ are false and $N(\bar{p})$ is false otherwise.

Wittgenstein believes that through this operation he can build up all propositions. Let us see how far we can do this. Let $(\bar{p}) = (p,q)$, and define $p/q \overset{df}{=} N(\bar{p})$, and we can immediately generate all the propositions that can be generated by Sheffers binary truth operation. We can also define quantification. Set $(\bar{\xi})$ be the class of all propositions which are obtained from "R (a,x)" by the proper substitution of names for the variable "x", then define

$$(\exists x)\ R\ (a,x) \overset{df}{=} N(N(\bar{\xi})), \quad \text{and } (X)\ R\ (a,x) \overset{df}{=} \sim(\exists x)\sim R(a,x).$$

(x)

5 Facts

Wittfenstein [g] wants any proposition to be true because of some fact, just as any elementary proposition is true because of some atomic fact. Any proposition's truth depends upon the truth of elementary propositions, whose truth depend upon the existence and non-existence of the atomic facts they assert to exist. Hence, Wittgenstein defines a positive fact to be the existence of atomic facts, and a negative fact to be the non-existence of atomic facts. A fact--a portion of reality -is the existence and non-existence of atomic facts.[14] Now any proposition will be true if there is fact which is the existence and non-existence of atomic facts, such that the elementary propositions which are accordingly true or false are combined to make the propositions true when they have these truth values.

6 Internal and External Properties

The confusion about internal and external properties in most philosophers work is caused by their belief that all propositions are of the subject predicate form. F.H. Bradley, a philosopher who only believed in the subject-predicate form, say's in Appearance and Reality, "That which exists in a whole has external relations. Whatever it fails to include within its own nature, must be related to it by the whole, and related externally. Now these extrensic relations, on the other hand, cannot do so. For a relation must at both ends affect, and pass into, the being of its terms. And hence the inner essence of what is finite itself both is, and is not, the relations which limit it."[15] Moore , Russell, and Wittgenstein deny that a relation has any effect at its ends. What something is, is not affected by the relationships it happens to have with other things. What a thing is depends only upon its internal properties. Wittgenstein thinks that the internal properties of a thing are what make it possible for it to occur in an atomic fact, the possibility of this must have already been in it.[16] Thus the properties (I will speak of properties and relations instead of internal and external properties or internal and external relations) of a thing determine all the atomic facts whichhit may occur in and that no others are possible. In addition, I think, Wittgenstein thinks this is all that properties can be a determine. Thus, he is dis- Or agreeing with a view expressed by Moore in Principia Ethica that goodness is a quality (or property) of things "'Good', then, if we mean by it that quality which we assert to a thing, when we say that it is good, is incapable of any definition, in the most important sense of that word."[17] In § 3, I said that if "R(a,b)" is an elementary proposition, "(x,y) R (x,y)" is the proposition that says that whenever the names of two appropriate simples are substituted for "x" and "y" in "R(x,y)", a true elementary proposition results; i.e. "(x,y) R (x,y)" is true if all elementary propositions with the same logical form as "R(a,b)" are true. The question now is what simples are appropriate. This is one of the things logical syntax would tell us, if we knew it. I want to discuss it from another point of view however. Suppose a,b,c, and d are simples whose properties we know. Suppose a and b may occur in some atomic fact, which the proportion "R(a,b)" asserts. Suppose c and d may not occur in this structure, then "R(c,d)" is not a false proposition, but nonsense.

7 Logical Space.

Suppose that all objects were given to us. Then, from the forms of the objects, we would know all the atomic facts which could occur. From this, we could determine all possible existences and non-existences of atomic facts, i.e. all facts possible. This space of all facts Wittgenstein calls logical space. All facts are in logical space, and, in addition all facts which are possible, but which do not happen to be, are in logical space. Logic is concerned with everything in logical space, not just facts.[18] A situation in logical space is a possible fact. Every proposition represents a situation in logical space.[19] A proposition is true if this situation in logical space is in the world - is a fact - and false otherwise.[20]

8 Wittgenstein's Elimination of Identy

Wittgenstein objects to Russell's definition of identity, since there is no reason that different objects could not have all their properties in common. It would still seem to me to be significant to say that two objects have all their properties in common, but this is not a sufficient condition for their identity.

Wittgenstein thinks that the equality sign is not needed at all in logic and all that is needed is the right usage of variables. He thinks that two different variables cannot take on the same value at once; e.g. "(x,y) R (x,y)" does not imply "R(a,a)" where a is a simple. He never seems to have recognized that "R(a,a)" is nonsense. Through the use of variables, he thinks identity can be elimated, and he gives a few this examples.[21]

9 Problems with Wittgenstein's Elimination of Identity and an Alternative.

There are certain problems in Wittgenstein's elimination of identity. observe the following: "(x,y,z) (R(x,y). S(x,z))"does not imply "R(a,b).S(a,b)", where a and b are appropriate simples, since "b" was substituted for both "y" and "z". Observe also that "R(a,b). S(a,b)" does make sense. Now, notice the following, from "(x,y,z)(R(x,y).S(x,z))" we should be able to infer "(z)(R(a,b).S(a,z))". Now, from this, we should be able to infer "R(a,b).S(a,b)", unless there is some additional notation introduced to indicate what variables have been replaced by what along the way.[22] The addition of such notation would probably somehow involve identity. For this reason, I do not think Wittgenstein's use of variables is quite satisfactory. I think, that when a number of variables occur in a single elementary propositional function, then any two or more of them cannot be replaced by the same thing, otherwise the result would be nonsense. However, if two variables only occur in different elementary propositional functions, then they may be replaced by the same thing, if it is appropriate in both places. E.g., from "(x,y)R(x,y)" we can infer ~~"R(a,a)"~~ "R(a,b)" but not "R(a,a)", in agreement with Wittgenstein. Notice another difficulty in Wittgenstein's notation. From "(x,y)R(x,y)" we should be able to infer "(y)R(a,y)," but, unless some notation was added to this, we can, without contradicting Wittgenstein's use of variables, infer "R(a,a)", which is nonsense. Also notice that from

"$(x,y,z)\ (R(x,y).S(x,z))$" we can infer, according to the alternative "$R(a,b).S(a,b)$", which does make sense, and which is elimiated by Wittgenstein's method. The use of variables I have proposed here seems to me to be the correct one, but I have been unable to eliminate identity through it, as might seem desirable.

10 My Definition of Identity

It seems to me that it is a sufficient condition of identity that two objects have all their relations (external properties) in common. I will define identity and diversity as follows. Let a and b be two simples, and let $(\bar{p})$ be the class of all propositions of one of the forms $\sim(R_2(a,c) \equiv R_2(b,c))$, $\sim(R_2(c,a) \equiv R_2(c,b))$, $\sim(R_3(a,c,d) \equiv R_3(b,c,d))$, $\sim(R_3(c,a,d) = R_3(c,b,d))$, $\sim(R_3(c,d,a) \equiv R_3(c,d,b))$..., for all relations R_2, R_3 .... and all appropriate simples c,d,.... Now I define:

$$a = b \overset{df}{=} N(\bar{p})$$

and

$$a \neq b \overset{df}{=} N(N(\bar{p}))$$

Now, a statement of identity or diversity so defined will only make sense if the two simples have the same form to start with otherwise the result is nonsense. We generally do not want to say things are identical anyway.

In the ideal language Wittgenstein proposes, every name denotes something different so we would know from the names themselves whether or not a statement of equality were true. Identity is useful however when it is turned into a propositional function. This will become apparent in the next section.

11 Russell's Theory of Descriptions

Expressions such as "the golden mountain" have caused philosophers many headaches. To say "the golden mountain does not exist," it would seem that there would have to be something which is the golden mountain, which, however, does not exist. Before inventing his theory of description, Russell proposed the following in <u>The Principles of Mathematics</u> (1903): "Being is that which belongs to every conceivable term, to every possible object of thought-in short/ to everything that can possiblely occur in any proposition ~~tion~~, true or false, and to all such propositions themselves. Being belongs to whatever can be counted. If A be any term that can be counted as one, it is plain that A is something, and therefore that A is. Thus unless 'A is not' be an empty sound, it must be false - whatever A may be, it certainly is. Numbers, the Homeric gods, relations, chimeras, and four-dimentional spaces all have being, for if they were not entities of a kind, we could make no propositions about them. Thus being is a general attribute of everything and to mention anything is to show that it is.

"Existence, on the contrary, is the prerogative of some only amoungst beings. To exist is to have a specific relation to existence - a relation, by the way which existence itself does not have........, what does not exist must be something, or it would be meaningless to deny its existence; and hence we need the concept of being as that which belongs even to the non-existent."[23] Thus, according to this theory, even round squares *are*, while none exist.

Russell soon came to reject this theory, the new theory appeared in his article "On Denoting" in <u>Mind</u> (1905). He had found a way to analize all propositions in which a description, such as "the golden mountain", would seem to be a subject into a proposition in which there is no such subject. The proposition "The golden mountain does not exist" becomes "There is no x such that x is a golden mountain and such that for any y, y is a golden mountain implies x = y." "x is a golden mountain" is analized to be "x is golden and x is mountainous." Notice that if there are two or more golden mountain mountains, the golden mountain will not exist. The golden mountain will exist only if there is one and only one golden mountain.

This theory struck the editor of Mind as so preposterous that he begged Russell not to have it published. Russell, however, was convinced of its correctness and had it published. It has been considered by many to be his most important contribution to logic.[24]

I will now give a simplified version of the notation defined in Principia Mathematica for descriptions. Let f(x) be any propositional function in which only x is a variable. It could be, for examples R(a,x), or (S(a,x,c). T(x,d)) v R (a,x). The notation "(7x)(f(x))" is read "the x such that f(x)" The symbol "(7x)(f(x))" is not itself defined. However, its uses are. The proposition "(7x) (f(x)) exists" is defined.

$$E!(7x)(f(x)) \overset{df}{=} (\exists y)\ (x)\ (f(y).\ f(x) \supset (x = y)).$$

Letting g(x) be a propositional function, g((7x) (f(x)) is defined as follows :

$$g((7x)(f(x)) \overset{df}{=} (\exists y)(x)\ (f(y).\ f(x) \supset (x=y).\ g(y))$$ [25]

It is for such uses as are indicated above that identity is needed in logic and mathematics. Russell said in his article "On Denoting" , "The usefulness of identity is explained by the above theory. No one outside a logic-book ever wishes to say 'x is x', and yet assertions of identity are often made in such forms as 'Scott was the author of Waverley' or 'thou art the man.' The meaning of such propositions cannot be stated without the notion of identity, although they are not simply statements that Scott is identical with another term, the author of Waverly, or that thou art identical with another term, the man. The shortest statement of 'Scott is the author of Waverley' seems to be Scott wrote Waverley; and it is always true of y that if y wrote Waverley, y is identical with Scott." It is in this way that identity enters into 'Scott is the author of Waverley; and it is owing to such uses that identity is worth offering."[26]

It seems to me that my definition of identity provided for all such uses, and that it is consistent with Wittgenstein's general propositional form.

12 Frege's Definition of Numbers

Over the centuries, the meaning of numbers has caused much puzzlement. The generally accepted definition of numbers was just given by Frege in 1884 in his Grundlagen der Arithmetik, which was almost completely ignored. It was discovered independently by Russel in 1901.[27]

I will briefly outline this definition, in an informal manner. Zero is defined as the class of all no-membered classes; namely the class of the null class. To say that there are zero persons in a room is to say that the class of all peoples in the room is a member of the class of all no-membered classes. Similarly, one is defined as the class of all one-membered classes; two is defined as the class of all two-membered classes; etc. Naturally, no membered class, one-membered class, two-membered class, can all be defined without reference to the numbers zero, one, or two. A rigorous development of the rational and real numbers as well as the natural numbers is given by Quine in Mathematical Logic.[28] I have discussed them here in order to show why the idea of classes of classes was important to Frege and Russell.

13 Russell's Paradox

In 1901, Russell discovered a paradox which threatened to destroy much of classical mathematics.[29] Some classes, it would seem, are members of themselves, e.g. the class of everything, that is not a dog, is not a dog; so it sould be a member of itself. On the other hand, it would seem that some classes are not members of themselves, e.g. the class of all dogs is not a dog; so it would not be a member of itself. Russeill was led to consider the class of all classes which are not members of themselves. Is it or is it not a member of itself? If it is a member of itself, then, by definition, it is not a member of itself. If it is not a member of itself, then by definition it is a member of itself.

In 1902, Russell wrote to Frege informing him of the paradox.[30] Frege replied promptly, saying in part: "Your discovery of the contradiction caused me the greatest surprize and, I would almost say, consternation, since it has shaken the basis on which I intended to build my arithmetic.... It is all the more serious since, ... not only the foundation of my arithmetic, but also the sole possible foundation of arithmetic seems to vanish." [31]

Russell considered many possible solutions to the paradox. The one which was used in Principia Mathematica was the ramified theory of types. Because of the way quantification is defined by Wittgenstein, the ramification is not needed, and hence neither is the axiom of Reducitility, which was ~~sued~~ used to cancel the ramification, making the whole thing silly anyway. [32]

First of all, Russell observed that there are, indeed, an infinite number of contradictions of the same kind. It turns out, in addition, that classes can be defined as incomplete symbols, as descriptions were. Then, if the other paradoxes can be eliminated, Russell's paradox also vanishes.

The simplest of these paradoxes occur when someone says, "I am lying."[33] If he is lying, he is telling the truth. If he is telling the truth, he is lying. Now, according to Russell's theory of types, in any propositional function, the variable has a range of significance. When the variable takes on one of the values in its range of significance, a true or false proposition results, otherwise the nonsense results. This differs from the theory previously presented about the appropriate value of variables. The theory of types was prior to it and probably was very suggestive to Wittgenstein. Logical types are the ranges of significance of various variables. Objects or simples form the first logical type. Propositions about simples form the second logical type, etc. Propositions about simples are called first order propositions. Propositions about first order propositions are called second order propositions, etc. Now suppose someone says, "I am asserting a first order proposition, which is false." This is a second order proposition, so he is not asserting a first order proposition at all. Hence he is lying, and the paradox vanishes. The solution, as I have outlined it, is the so called simple theory of types. [34]

15 Wittgenstein's Rejection of the Theory of Types

Wittgenstein briefly rejects the Theory of the Types in the Tractatus. [35] I do not see how the expression "F(Øu) " can have any meaning to Wittgenstein in view of other of his remarks. It would seem that in it he is saying there can indeed be relations of relations expressed in language, although a relations of relations cannot be defined circularly. I believe Wittgenstein reputes this in his mysticism.

I will quote some passage from his notebooks, which I think must have more closely reflected his thought.

"This throws light on what we say in grammar: 'One word refers to another.'

"That is why the point in the above cases is to say how propositions hang together internally. How the propositional bond comes into existence. [cf. 4.221]

"How can a function refer to a proposition?? Always the old old questions.

"Don't let yourself get overwhelmed with questions; just take it easy.

"'Ø (yx)': Suppose we are given a function of a subject-predicate proposition
'Ø(ψx)'
and we try to explain the way the function refers to the proposition by saying: The function only relates immediately to the subject of the subject-predicate proposition, and what signifies is the logical product of this relation and the subject-predicate propositional sign. Now if we say this, it can be asked: If you can explain the proposition like that, why not give an anlogous explanation of what it stands for? Namely: 'It is not a function of a subject-predicate fact but the logical product of such a fact and of a function of its subject'? Must not the objection to the latter expression hold against the former too?" [36]

The analysis he defines and rejects seems to be:

'Ø(yx) = Xx ·Xx' for some X for any Ø. I ~~can~~ am not certain why he rejected

$\varnothing(\psi x) = \chi x \cdot \psi y$

it, but there are obvious reasons it is inadequate. ~~Set~~ Let 'Ø(Xx)' be 'Xx' is false,' then, if this analysis is correct, 'Xx' must be true. If it were false, then, by the definition of 'Ø(Xx)' it would have to be true. Therefore the analysis is wrong, or at least many functions of functions we would like to analyze cannot be analyzed by it.

Wittgenstein's conclusion seems to be that functions of functions are impossible, that what is true of functions is shown by them, and cannot be said about them. He says in his notebooks:

"How is it reconcilable with the task of philosophy, that logic should take care of itself? If, for example, we ask: Is such and such a fact of the subject-predicate form? We must surely know what we mean by 'subject-preidcate form.' We must know whether there is such a form at all. How can we know this? 'From the signs.' But how? For we haven't got any signs of this form. We may indeed say: We have signs that behave like signs of the subject-preidcate form, but does that mean that there really must be facts of this form? That is, when those signs are completely analyzed? And here the question arises again:

Does such a complete analysis exist? And if not: then what is the task of philosophy?!!? "Then can we ask ourselves: Does the subject -predicate form exist? Does the relational form exist? Do any of the forms exist at all that Russell and I were always talking about? (Russell would say: 'Yes! that's self evident.' Well!)

"Then : if everything that needs to be shown is shown by the existence of subject-predicate sentences etc., the task of philosophy is different from what I originally supposed. But if that is not how it is, then what is lacking would have to be shown by means of some kind of experience, and that I regard as out of the question." 37

16 Wittgenstein's Mysticism

I think that, for Wittgenstein, a person is his world, i.e. the totality of the facts, which are his thoughts. [38] Any thought is a fact, though all facts need not be thoughts.[39] Now, since the I is a totality of facts, only the propositions corresponding tothose thoughts will occur in a person's language. A person, hence, cannot say anything about himself , only about his thoughts. To talk about himself he would have to have a language about language, as in the theory of types, which Wittgenstein has rejected. The I is to a person's thoughts as an eye is to its visual field. An eye cannot see itself. A person cannot think about himself, he can only think thoughts, which he is the totality of .[40]

As a result of this, there can ~~be not~~ not be ethical or aesthetic propositions and the rest of Wittgenstein's mysticism [41] follows.

17 Hierarchies of Languages

In his introduction to Tractatus, Russell suggested some sort of hierarchy of language mught be a way to avoid Wittgenstein's Mysticism.[42] Russell carries this out this in his Inquiry into Meaning and Truth.[43] Most logicians since Wittgenstein have adopted some sort of hierarchy as without it, most of mathematics becomes impossible. The logician says, if we do not assume this, we cannot develop mathematics. Therefore this is true. Wittgenstein's objection to such procedures seems to me to be final. "Logic must take care of itself."[44] Unless we can develop mathematics without such devices, mathematics must, logically, fall.

18 A Solution of the Paradox

I am convinced that in the following manner, any paradoxes can be avoided. The extent to which mathematics may be deduced remains to be seen. I define classes of objects, as incomplete symbols, in the following manner:

$$q \in \hat{x}\, f(x) \overset{df}{=} f(a).$$

We need a way to define classes of classes, i.e. functions of functions. This whould be possible if we can devise a way to talk about language in language. This is possible in so far as any proposition is a fact, but not in so far as it is a projection of thought. There are propositions about propositions qua fact, not qua reflection of thought. Any particular occurence of the letter 'a' is a fact. It is a structure of simples. Thus there is a propositional function $f_a(x)$ for any occurence of the letter 'a' which is true when 'x' is replaced by the name of a simple in the particular occurence of 'a' and false if it is replaced by any other appropriate simple. In addition, there is a propositional function $g_a(x)$ which is true if 'x' is replaced by a name of a simple in any occurence of the letter 'a'. Given these two functions, any occurence of the letter 'a' should be definable as an incomplete symbol. Set $f_b(x)$ and $g_a(x)$ be similarly defined for the letter 'b'.

The particular occurence of the letter 'a' is defined as $\hat{w}\, f_a(w)$. The particular occurence of the letter 'b' is defined as $\hat{x}\, f_b(x)$.

Then $R(\hat{w}\, f_a(w), \hat{x}\, f_b(x)) = (y)\,(z)(y \in \hat{w}\, f_a(w) . z \in \hat{x}\, f_b(x) \supset g(y, z))$ for some $g(y,z)$

Now define:

$$\hat{x}\, f(x) \in \hat{a}\, R(a, \hat{y}\, g(y)) = R(\hat{x}\, f(x), \hat{y}\, g(y))$$

This defines a classes of classes of objects, as an incomplete symbol. The method I have just indicated can be extended indefinitely. When done so, I think it would be a great formal similarity to Russell's theory of types, but without the same interpretation. It would not be a hierarchy of language, as Russell suggested. As in it one language could talk about the meanings of the words in another.

"In the secondary language, we are concerned with the words of the object-language, not simply as noises or bodily movements, for in that respect they belong to the object language, but as have meanings."[45]

Since I have not defined such a hierarchy as Russell suggests, most of Wittgenstein's mysticism would remain intact.

NOTES

[1] Ludwig Wittgenstein, Tractatus Logico- Philosophicus, (New York, 1969) 2.01, 2.032

[2] Ibid, 2.061

[3] Ibid, 2.04

[4] Ibid, 2.0141, 2.033

[5] Gottlob Frege, Begriffsschrift in From Frege to Gödel, ed., Jean van Heijlenoort (Cambridge, 1967), p 7

[6] Wittgenstein, Tractatus, 3. 432

[7] Alfred North Whitehead and Bertrand Russell, Principia Mathematica to *56 (Cambridge, 1967), p. xv.

[8] Wittgenstein, Tractatus, 4.2211

[9] Henry Maurice Sheffer, "a Set of Five Independent Postulates for Boolean begebras, with application to Logical Constants," Trans. American Math. Soc., Vol. 14, (1913),, pp 481-488

[10] Willard Van Orman Quine, Mathemmatical Logic (New York, 1962) pp 45-49

[11] Russell, "Introduction" in Tractatus, p. xv.

[12] Wittgenstein, Tractatus, 5.2522

[13] Ibid, 5.501, 5.502

[14] Ibid, 2.06

[15] F.H. Bradley, Appearance and Reality, (London, 1969), p.322

[16] Wittgenstein, Tractatus, 2.011, 2.0124

[17] George Edward Moore, Principia Ethica, (Cambridge, 1954), p.9

[18] Wittgenstein, Tractatus, 2.0121

[19] Ibid, 2.11

[20] Ibid, 1.13

[21] Ibid, 5.531-5.5321

[22] Rudolf ~~Lamp~~ Carnap, The Logical Syntax of Language (Patterson, New Jersey, 1959) pp.49-51

[23] Russell, The Principles of Mathematics (New York), pp 449-450

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[24]Russell, My Philosophical Development (New York, 1959), pp.83-84

[25]Whitehead and Russell, Principia, pp.66-68

[26]Russell, "On Denoting," in Logic and Knowledge (London, 1966)p.55

[27]Russell, Introduction toMathematical Philosophy (London, 1920), p.11

[28]Quine, Mathematical Logic, pp. 237-279

[29]Russell, Development, pp.75-76

[30]Russell, "Letter to Frege" in From Frege to Gödel , pp. 124-215

[31]Frege, Letter to Russell" in From Frege to Gödel, pp. 127-128

[32]Quine From Frege to Gödel, pp 150-152

[33]Epistle to Titus: I, 12-13

[34]Carnap, pp. 84-87

[35]Wittgenstein, Tractatus , 3.33-3.333

[36]Witthenstein, Notebooks 1914-1916 (New York, 1961), pp.5e-6e

[37]Ibid, pp. 2e-3e

[38]Wittgenstein, Tractatus, 5.63

[39]Ibid, 2.141,3

[40]Ibid , 5.6331

[41]Ibid, 6.4-7

[42]Russell,"Introduction", p.xxii

[43]Russell, An Inquiry into Meaning and Truth (London, 1961) pp.62-93

[44]Wittgenstein, Notebooks, p. 2e

[45]Russell, Inquiry, p.78

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