The Philosophy of Organism

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Part I

§1 Eternal Objects

This section sets forth the structure of the realm of eternal objects, without regard to their content. The eternal objects are the universals in Whitehead's system. They are repeated throughout experience. The actual entities, the particular occasions of experience, are never repeated. The lowest grade of eternal objects, which I will call the 1st grade, consists of the eternal objects which do not presuppose any other eternal objects, but have direct ingression into actual occasions. The 2nd grade of eternal objects consists of those eternal objects which have as relata only eternal objects of the 1st grade. The n^{th} grade of eternal objects consist of those eternal objects consists of those eternal objects which have as relata only eternal objects of the $n-1^{th}$, $n-2^{th}$, ..., and 1st grades. 1

An actual occasion consists of two poles. A mental pole and a physical pole. The mental pole consists of a number of finite abstractive hierarchies; the physical pole consists of one infinite abstractive hierarchy. 2

"An 'abstractive hierarchy based upon g,' where g is a group of simple eternal objects, is a set of eternal objects which satisfy the following conditions,

(i) the members of g belong to it, and are the only simple eternal objects in the hierarchy,
(ii) the components of any complex eternal object in the hierarchy are also in the hierarchy, and
(iii) any set of eternal belonging to the hierarchy, whether all of the same grade or whether differing among themselves as to grade, are jointly among the components or derivative

components of at least one eternal object which also belongs to the hierarchy."[3]

Finite abstractive hierarchies stop at a finite grade of complexity, infinite abstractive hierarchies have members of every grade of complexity.[4] The physical pole has as its origin the simple physical feelings of the actual occasion. The mental pole has as its origin the simple conceptual feelings of the actual occasion.

§2 Relations and General Principles

Relations and general principles are are specific sorts of eternal objects. "A 'relation' between occasions is an eternal object illustrated in the complex of mutual prehensions by virtue of which those occasions constitute a nexus."[5] Suppose an occasion S prehends occasions A, B, and C. A, B, and C are then together in S in some manner, which is an eternal object, say ϕ. Then relative to occasion S, $\phi(A, B, C)$. As simple eternal objects are qualitative, it is necessary that relations be at least of the 2nd grade.

Relations are defined to be dual relations, triple relations, etc. respectively as the nexus concerned contain two, three, etc. members.[6]

General principles are eternal objects whose instances are eternal objects and whose instances are mutually exclusive. Color is an example of a general principle.[7] General principles, by definition, must also be at least of the 2nd grade.

§3 Indicative systems of dual Relations

Every proposition presupposes an indicative relational system in some general nexus.[8] Thus indicative relational systems are very important. "A nexus exhibits an 'indicative system' of dual relations among its members, when

(i) one, and only one, relation of the system relates each pair of its members; and

(ii) these relations are instances of a general principle, and

(iii) the relation (in the system) between any member A and any other member B does not also relate A and a member of the nexus other than B, and

(iv) the relations (in the system) between A and B and between A and C suffice to define the relation (in the system) between B and C, where A, B, and C are any three members of the nexus."[9]

The following is an example of an indicative relational system among actual entities A, B, C, D, E F, G, H, I. The relations are $R_{m,n}$ where m and n are integers between -2 and 2, inclusive. The following relations hold: $R_{1,0}(A,B)$, $R_{2,0}(A,C)$, $R_{0,-1}(A,D)$, $R_{1,-1}(A,E)$, $R_{2,-1}(A,F)$, $R_{0,-2}(A,G)$, $R_{1,-2}(A,H)$, and $R_{2,-2}(A,I)$. In accordance with (iv) above all the other relations in the system between members of this nexus are derivable from above.

$$R_{m,n}(X,Y) \cdot R_{i,j}(X,Z) . \supset R_{i-m,j-n}(Y,Z)$$

It is to be noted that the use of integers as subscripts is merely a convenience

§4 Propositions

A proposition is the possibility of actual occasions in the actual world of some actual occasion fulfilling some eternal object.

Whitehead defines two kinds of propositions.

"A 'singular proposition' is the potentiality of an actual world including a definite set of actual entities in a nexus of reaction involving the hypothetical ingression of a definite set of eternal objects."[10]

"A 'general proposition' only differs from a 'singular' proposition by the generalization of 'one definite set of actual entities into any set belonging to a certain sort of sets.' If the sort of sets includes all sets with potentiality for that nexus of reaction, the proposition is called 'universal'."[11]

A proposition which is the possibility of the ingression of the eternal object R, into actual occasions A, B, and C is symbolized R(A, B, C). The universal proposition which is the possibility of the ingression of R into all sets with that possibility ~~[illegible]~~ is symbolized (X, Y, Z) R(X, Y, Z). Also if the incompatibility of propositions is introduced, then the notions of 'and', 'or', 'not', 'if ... then', 'if and only if', etc. are definable.

The set of actual occasions involved in the proposition are the logical subjects of the proposition. The eternal objects involved are the predicates of the proposition. One complex eternal object is formed by the predicates; this is the complex predicate.[12]

The locus of a proposition is the set of all actual occasions whose actual worlds include the logical subjects of the proposition.[13]
If the locus of a proposition includes as a member a particular actual occasion, then the proposition is true or false with regard to that occasion. If it is realized in the actual world of that occasion, it is true; if it is not realized, it is false.[14]

As mentioned before, Whitehead says every proposition presupposes an indicative system of relations. It will now be shown how such an indicative system would be used. The proposition in question will be about actual occasions in the actual world of A. Its actual world includes occasions B, C, D, E, F, G, H, I for which the indicative system of dual relations in §3 holds.

The indicative system of relations first allows equality to be defined for that actual occasion:

$x = y \overset{df}{=} (\exists R_{ij}) R_{ij}(A, x) . R_{ij}(A, y)$, i.e. two actual occasions in the world of A are identical if some member of the indicative system of dual relations relates them both to A.

Next descriptions can be defined as in <u>Principia Mathematica</u>

$$\psi(\iota x)(\phi(x)) \overset{df}{=} : (\exists y) . \phi x . \equiv_x . x = y : \psi y$$ [15]

Then any occasion in the nexus of occasions B, C, D, E, F, G, H, and I can be refered to descriptively with the indicative system. The references are

$$(\iota x)\,(R_{1\text{-}0}\,(A, x))\quad\text{for } B$$
$$(\iota x)\,(R_{2\text{-}0}\,(A, x))\quad\text{for } C$$
$$(\iota x)\,(R_{0\text{-}1}\,(A, x))\quad\text{for } D$$
$$(\iota x)\,(R_{1\text{-}1}\,(A, x))\quad\text{for } E$$
$$(\iota x)\,(R_{2\text{-}1}\,(A, x))\quad\text{for } F$$
$$(\iota x)\,(R_{0\text{-}2}\,(A, x))\quad\text{for } G$$
$$(\iota x)\,(R_{1\text{-}2}\,(A, x))\quad\text{for } H$$
$$(\iota x)\,(R_{2\text{-}2}\,(A, x))\quad\text{for } I$$

Thus the only 'name' required in the expression of a proposition is the name of the occasion in which the proposition is located.

Part II

§1 The Modes of Objectification

In Whitehead's system, there are two ways actual entities are objectified in a given actual entity. They are the modes of causal objectification and the mode of presentational objectification. Causal objectification is the primary mode. Presentational objectification is a derivative mode.

Causal objectification is the transmission of feeling from one actual occasion to another actual occasion. "In '*causal objectification*' what is felt subjectively by the objectified actual entity is transmitted objectively to the concrescent actualities which supersede it." [1]

The *past* of an actual occasion is the set of all actual occasions which are causally objectified in it. The *future* of an actual occasion is the set of all actual occasions in which it is causally objectified. Two actual occasions are *contemporaries* if neither is in the past of the other. A *duration* is a set of actual occasions such that (1) all actual occasions in the set are contemporaries of each other and (2) no more actual occasions can be added to the set without losing property (1). [2]

By definition, actual occasions which are contemporaries cannot be causally objectified in each other. However, they do bear special relationships to each other, in that they arise from pasts which are in part the same. In this sense, they are objectified in each other. This kind of objectification is termed *presentational objectification*. "In 'presentational objectification,' the relational eternal objects fall into two sets, one set contributed by the 'extensive' perspective of the perceived from the position of the perceiver, and the other set by the antecedent concrescent phases of the perceiver."[3]

Any actual occasion will lie in an infinite number of durations. According to Whitehead it bears a special relation to one of these durations. All the actual occasions presentationally objectified in the given actual occasion will lie in this duration. This duration is called the <u>presented duration</u> of the actual occasion[4], and the actual occasion is said to have <u>cogredience</u> with this duration.[5]

The extensive continuum is based upon the presentational objectification of the contemporary world. Contemporary occasions are causally independent. They have interrelationships because they arise from a past common at least in parts. Whitehead derives the extensive continuum from the peculiar character of the relationships between these causally independent occasions.

"The mutual independence of contemporary occasions lies strictly within the sphere of their teleological self-creation. The occasions originate from a common past and their objective immortality operates within a common future. Thus indirectly, via the immanence of the past and the immanence of the future, the occasions are connected. But the immediate activity of self creation is separate and private, so far as contemporaries are concerned.

"There is thus a certain indirect immanence of contemporary occasions in each other. For if A and B be contemporaries, and C be in the past of both of them, then A and B are each in a sense immanent in C, in the way the future can be immanent in the past. But C is objectively immortal in both A and B. Thus, in this indirect sense, A is immanent in B and B is immanent in A."[6]

According to Whitehead there may be consciousness of both forms of objectification. However consciousness of presentational objectification is more vivid, and it is what is ordinarily termed perception. Thus, since actual occasions are causally independent, Whitehead uses this to explain why Hume was unable to find any perception of causation.

In another way the causal independence of contemporaries is handy. Because of it, the contemporary world is presented as an extensive continuum. "By reason of the principle of contemporary independence, the contemporary world is objectified for us under the aspect of passive potentiality. The very sense-data by which its parts are differentiated by antecedent states of our own bodies, and so is their distribution in contemporary space. Our direct perception of the contemporary world is thus reduced to extension defining (i) our own geometrical perspectives, and (ii) possibilities of mutual perspectives for other contemporary entities inter se, and (iii) possibilities of division. These possibilities of division constitute the external world as a continuum."[7]

"The contemporary world as perceived by the senses is the datum for contemporary actuality and therefore continuous — divisible but not divided. The contemporary world is in fact divided and atomic, being a multiplicity of definite actual entities. These contemporary actual entities are divided from each other, and are not themselves divisible into other contemporary actual entities."[8]

§2 The Extensive Continuum

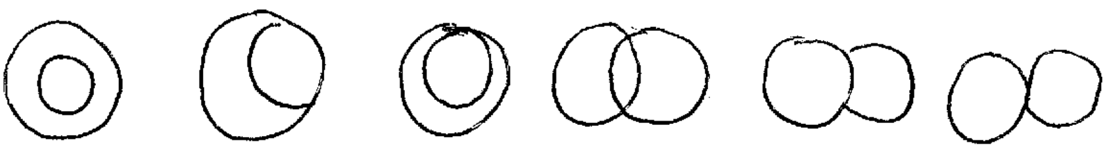
So far, Whitehead has established that the contemporary world is presentationally objectified as a continuous whole. However, he has not established any continuous relations between an actual occasion and the occasions in its past or in its future. [redacted] Either such [redacted] relations can be defined in terms of the relation established, or such relations must be assumed to exist, as a generalization from the established relations. As Whitehead gives no such definitions he seems to use the method of generalization. This is in agreement with the methodology for speculative philosophy he outlines in the first chapter of *Process and Reality* although to me (and probably to him) definitions would be preferable. Whether such definitions could be made is a question to which I have no answer. There can be no doubt, however, that Whitehead says all occasions are related in one extensive continuum. "The real potentialities relative to all standpoints are coordinated as diverse determinations of one extensive continuum. This extensive continuum is one relational complex in which all potential objectifications find their niche. It underlies the whole world, past, present, future."

Every actual occasion has a region which its concrescence presupposes. These regions make up the extensive continuum. "The quantum is the extensive region. This region is the determinate basis which the concrescence presupposes. This basis governs the objectifications of the actual world which are possible for the novel concrescence. The coordinate divisibility of the satisfaction is the 'satisfaction' considered in its relationship to the divisibility of the region."[10]

The division of a region is based upon two things. First, every actual occasion has a region as its standpoint. Second, the regions of actual occasions presentationally objectified in a given actual occasion are not distinguishable, but are objectified as a continuous whole. Thus ~~any~~, in this respect, any region can be taken to be the region of some actual occasion. The proposition that this is the case, is strictly speaking false, but so far as the physical relations of actual occasions are concerned, it is true. "A coordinate division is thus to be classed as a generic contrast. The two components of the contrast are, (i) the ~~[illegible]~~ parent actual entity, (ii) the proposition which is the potentiality of that superject having arisen from the physical standpoint of the restricted sub-region. The proposition is thus the potentiality of eliminating from the physical pole of the parent entity all the objectified actual world, except those ~~[illegible]~~ elements derivable from that standpoint, and yet retaining the relevant elements of the subjective form."[11]

§3 Extensive Connection

Whitehead uses *extensive connection* as a binary relation between regions. It is an undefined or primitive relation, but the following diagrams are given to illustrate it.[12]



The following definitions are made in terms of extensive connection.[13]

Region A *includes* region B if every region connected with B is also connected with A.

Two regions *overlap* if there is a third region they both include.

A *dissection* of any given region A is a set of regions, which is ~~[illegible]~~ such that

(i) all its members are included in A,

(ii) no two of its members overlap

(iii) any region included in A, but not a member of the set, either is in one member of the set, or overlaps more than one member of the set.

A region is an *intersect* of two overlapping regions, A and B, if

(i) either it is included in both A and B, or it is one of the two regions and is included in the other, and

(ii) no region, also included in both A and B, can overlap it without being included in it.

Two regions intersect with *unique intersection*, if they have one and only one intersection.

Two regions ~~[illegible]~~ intersect with *multiple intersection* if they have more than one intersection.

Two regions are *externally connected* if
(i) they are connected, and
(ii) they do not overlap.

A region B is *tangentially included* in a region A if
(i) B is included in A, and
(ii) there are regions which are externally connected with both A and B

A region B is *non-tangentially included* in a region A if
(i) B is included in A, and
(ii) B is not tangentially included in A

The following definitions can also be made.[14]

Two regions A and B have *junction* if there is a third region C such that
(i) both A and B are included in C, and
(ii) no region exists which is included in C and which overlaps neither A nor B.

Two regions have *adjunction*, if they have junction, and do not intersect

Two regions A and B have *injunction*, if A is included in B and there exist a region B, which does not intersect B, but has adjunction with A. Injunction appears to hold between region if and only if one is tangentially included in the other.

The remainder of this part of the paper will primarily be concerned with providing the definition necessary to state Whitehead's philosophy of science in terms of his later philosophy. The existence of ~~the~~ entities corresponding to the definitions is not assured by the later philosophy which is to ~~be~~ have complete metaphysical generality. However their meaning does fit within this complete generality. This task is useful for two reasons. It permitts a more complete understanding of the metaphysical foundation of Whiteheads philosophy of science, and it also gives an illustration of his later philosophy will can be extremely useful for the understanding of that philosophy and for the discussion of particular problems.

The main problem in this task is that the philosophy of science is stated in terms of events rather than regions. For the most part, they are the same. The main difference is that regions are necessarily finite, whereas events could be infinite in extent. Durations were a certain kind of event. Now they are a certain kind of set of events. Thus almost all definitions concerning durations must be restated.

The following list of axioms are assumed for the purposes of this part, to hold for regions. They do not have complete metaphysical generality, but are derived from axioms about events. The region a is included in a region b will be symbolized bKa. The axioms are:

i) If aKb, then $a \neq b$
ii) Every region includes other regions, and is included in other regions.
iii) K is transitive
iv) If aKc there are regions such as b where aKb and bKc.
v) If a and b are any regions, there are regions such as c where cKa and cKb.
vi) If the parts of b are also parts of a, and $a \neq b$, then aKb.
vii) Every region has adjunction with other regions.
viii) Every region has injunction with other regions.

§4 Extensive Abstraction

Extensive abstraction is the method used by Whitehead to define points, lines, etc from regions. Roughly speaking, a point is defined as the set of all sets of regions which would ordinarily be said to converge to that point. Of course, the definitions are made without reference to points, and the convergence requirements are stated purely in terms of regions.

The following definitions are made:[16]

A set of regions is an *abstractive set* if
(i) any two members of the set are such that one of them includes the other non-tangentially
(ii) there is no region included in every member of the set.

An abstractive set α *covers* an abstractive set β, if every member of the set α includes some members of the set β.

Two abstractive sets are *equivalent* if each set covers the other

A *geometrical element* is a complete group of abstractive sets equivalent to each other and not equivalent to any abstractive set outside the group.

A geometrical element is *associated* with an abstractive set, if the abstractive set is a member of the geometrical element.

A geometrical element a is *incident* in a geometrical element b, if every member of b covers every member of a, but a and b are not identical.

A *point* is a geometrical element which has no other geometrical element incident in it.

An abstractive set is *punctual* if it is a member of a point.

The members of a geometrical element are <u>prime in reference to assigned conditions</u>, if

(i) every member of that geometrical element satisfies those conditions

(ii) if any abstractive set satisfies those conditions, every member of its associated geometrical element satisfies them;

(iii) there is no geometrical element, with members satisfying those condition, which is also incident in the given geometrical element.

A <u>segment between point p and p</u> is a geometrical element prime in reference to the condition that the point p and q are incident in it.

p and q are called the <u>end points</u> of the segment between p and q.

An abstractive set is <u>segmental</u> if it is a member of a segment.

A point is <u>situated in a region</u> if the region is a member of one of the punctual abstractive sets which compose the point.

A point is <u>situated in the surface of a region</u> if all the regions in which it is situated overlap that region but are not included in it.

A <u>complete locus</u> is a set of points which compose either

(i) all the points situated in a region, or

(ii) all the points situated in the surface of a region, or

(iii) all the points incident in a geometrical element.

When a complete locus consists of all the points situated in a region, it is called the <u>volume</u> of that region; when a complete locus consists of all the points situated in the surface of a region, the locus itself is called the <u>surface</u> of the region; when a complete locus consists of all the points incident in a segment between endpoints, the locus is called the <u>linear stretch</u> between those end points.

§5 Durations and Moments

The definition of the past, future, and contemporaries of a region will be assumed to have been defined in exact analogy with the definition for an actual occasion. In a sense, this is unnecessary as a coordinate division involves as an element in its contrast the proposition that the region is an actual occasion, or rather its standpoint, so the original definition would really apply in any case. The definition of a duration will be repeated for convenience. A <u>duration</u> is a set of regions which are all contemporaries of each other, and which are such that all other regions are in the past or future of some member of the set.

Two durations, α and β, are <u>parallel</u> if there is a third duration γ such that every region in α or in β is included in some region in γ.[17]

A duration β is <u>non-tangentially included</u> in a duration α if every region in β is non tangentially included in some region of α

A <u>momental set</u> of durations is a set of durations such that of any two members of the set, one non-tangentially includes the other, and no duration is non-tangentially included in every member of the set

A momental set α <u>covers</u> a momental set β if every member of α nontangentially includes some member of β

Two momental sets are <u>equivalent</u> if each covers the other.

A <u>moment</u> is a complete group of momental sets equivalent to each other, and not equivalent to any momental sets outside their group.[18]

A momental set α <u>covers</u> an abstractive set β if every region in β is included is some region in some duration in α.

A geometrical element a is <u>incident</u> in a moment b if every member of b covers every member of a

A *family of parallel durations* is a set of durations all parallel to each other and not parallel to any duration outside the set.[19]

As a consequence of the definition of a moment and of a family of parallel durations, it turns out that all the durations in any momental set of a given moment all are in one family of parallel durations. Two moments are *parallel* if the durations in any momental set in either of the moments all lie in the same family of parallel moments. A moment is *parallel* to a duration (and vice versa) if any duration in any momental set in the duration is parallel to the duration.

A *time system* is a set of durations and moments all parallel to each other and not parallel to any durations or moments outside the set.[20]

A point *bounds* a duration if every region in which the point is situated intersects some region in the duration, and no region in which the point is situated is included in any region in the duration. A moment *bounds* (is a boundary of) a duration if every point incident in the moment bounds the duration.[21]

A moment is *situated in* a duration if the duration is in one of the momental sets in the moment. A moment b *lies between* moments a and c if b is situated in some region which a and c bound.[22]

The following axioms are assumed to hold:

(1) Every duration is bounded by two moments, and every pair of parallel moments bound one duration of that time system.

(2) The relation of lying between has the following properties which generate continuous serial order in each time system. They are:

(α) Of any three moments of the same time system, one of them lies between the other two

(β) If the moment b lies between moments a and c, and the moment c lies between moments b and d, then b lies between moments a and d.

(γ) There are not four moments in the same time system such that one of them lies between each pair of the remaining three.

(δ) The serial order among moments of the same time system has the Cantor-Dedekind type of continuity.

§6 Instants, Instantaneous Planes, and Instantaneous Straight Lines

An *instant* is the set of all points incident in a moment. An *instantaneous plane* (a level) is the set of all points incident in both of two moments which are not parallel. An *instantaneous straight line* (or sect) is a set of points such that

(i) there are three moments, no two of which are parallel and the set is the set of all points incident in all three of these moments.

(ii) the set of points is neither empty nor an instantaneous plane.

Three moments are *co-level* if all the points in some instantaneous plane are incident in all three. Three moments are *co-sect* if all the points in some instantaneous straight line are incident in all three, but the three are not co-level.[24] Instants, instantaneous planes, and instantaneous straight lines are *loci* in addition to the loci previously defined. A locus is *co-momental*, if all its points are in one instant. A locus is *vagrant* if it is not co-momental. Two points are *sequent* if there is no instant containing both.[25] A locus is *incident* in a moment, if all of its points are incident in the moment.

Two instantaneous planes a and b are *comomentally parallel* if they are incident in a moment c and there are two moments d and e, parallel to each other, and a is the set of points incident in both c and d and b is the set of points incident in both c and e.

Two instantaneous planes are *parallel* if there is a third instantaneous plane to which they are both co-momentally parallel.

Two instantaneous straight lines a and b are *co-momentally parallel* if they are subsets of an instantaneous plane c and there are instantaneous planes d and e parallel to each other and a is the intersection of c and d and b is the intersection of c and e. (intersection is here used in the sense of the intersection of two sets not two regions)

Two instantaneous straight lines are *parallel* if there is a third instantaneous straight line to which they are both co-momentally parallel. 26

§ 7 Cogredience

A region is <u>included</u> in a duration if it is included in some region in the duration. A region intersects a moment if some ~~[illegible]~~ situated in the region is incident in the moment. A region <u>extends throughout</u> a duration if the region is included in the duration and the region intersects any moment situated in the duration.[27]

Whitehead never gives a list of axioms for the cogredience of ~~[illegible]~~ regions in durations, but Palter gives the following properties which he says he deduces from Whitehead's use of the concept. The region a being cogredient in duration d is symbolized a G d. The properties are:

i) For every region a there is one and only one duration d such that a G d.

ii) If a G d there are ~~[illegible]~~ regions b such that a K b and b G d.

iii) If a G d, there is a region b such that b K a and b G d.

iv) If a G d, then a extends throughout d.[28]

§ 8 Stations.

An abstractive set is stationary with duration d at point p incident in d if it is prime with respect to the condition that each of its members ~~[illegible]~~ include regions which (i) p is situated in and (ii) ~~are~~ cogredient with d.

The station of p in d is the geometrical element associated with an abstractive set which is stationary with duration d at point p.[29]

The following important theorem can be proven: If d and d' are durations of the same time system such that d K d', and if the point p is incident in d' and s and s' are the station of p in d and d' respectively, then s covers s'.[30] Here d K d' if for every region σ' in d' there is a region σ in d such that σ K σ'.

A peculiar difficulty arises here. One of three things must happen. They are:

1) No point has a station in any duration,
2) The shapes of possible regions must be restricted.
3) Some regions do not have cogredience with any duration.

This is proven as follows. Around any point in any duration some weird shaped region ~~[illegible]~~ can be placed within that duration such that no 'straight' line extends through it the ~~[illegible]~~ region from one boundary of the duration to the other. If that region is cogredient in the duration, then no ~~[illegible]~~ station will exist for that point in that duration, as stations are to be used to define 'straight' lines which are not instantaneous.

The easiest way to deal with this problem is to say that some regions do not have cogredience in any duration. Such regions will not really have the potentiality of being the standpoints of actual ~~entities~~ occasions, as all actual occasions must be cogredient in some duration. Thus actual occasions will retain the possibility of being actual occasions only as regards their relations of extensive connection and the relations of one region being in the past, present, or future of another region. This rules out the first property given by Palter and stated in the last section.

If the above is correct, the field of the relation of cogredience will be restricted to a set of durations determined by their shape. Intuitively I think this would be the class of oval regions. If I am correct it would follow that every actual occasion has an oval region as its standpoint. This would also permit oval regions to be defined in terms of cogredience, which are only partly defined in a very complicated manner in Chapter 3 of Part 4 of <u>Process and Reality</u>.

§9 Sequential Straight Lines and Vagrant Planes.

A <u>sequential straight line</u> (or point track) is the set of all points incident in some station of a given point in duration of a given time-system.

Two sequential straight lines, a and b, are <u>parallel</u> if there is a time system such that there are points p and q and a is the set of all points incident in some station of p in some duration of the time system and b is the set of all points incident in some station of q in some duration of the time system.[31]

A <u>vagrant plane</u> (or matrix) is the set of all points m such that there exist an instantaneous straight line r, and a point p not co-momental with r such that ~~[illegible]~~ m is the set of all points which are either

(i) in some instaneous straight line including p and intersecting r, or

(ii) in some sequential straight line including p and intersecting r, or

(iii) in the instantaneous straight line through p and parallel to r,[32]

§10 Normality

§11 Triangles and Quadrilaterals

A segment is an instantaneous straight segment if all the points incident in it are in one instantaneous straight line. A segment is a sequential straight segment if all the points in it are incident in one sequential straight line. A segment is a straight segment if it is an instantaneous straight segment or a sequential straight segment. The straight segment between points A and B will be symbolized AB. A straight line is an instantaneous straight line or a sequential straight line. Two straight segments are parallel if the straight lines, in which the points incident in them are, are parallel. Two straight segments are normal if the straight lines, in which the points incident in them are, are normal.

A geometrical element is a triangle with vertices A, B, and C if it is prime with respect to the condition that AB, BC, and CA are incident in it, and no straight segment has A, B, and C all incident in it. The segments AB, BC and CA are the sides of the triangle. The altitude of a triangle is the geometrical element prime with respect to the condition that the vertex of the triangle not and endpoint of the given side is incident in it, it is normal to the given side, and some point incident in it is in the straight line in which the points of the given side are incident.

A geometrical element is a <u>quadrilateral with vertices</u> A, B, C, and D if it is prime in respect to these conditions:

i) it satisfies one of the following

α) the straight segments, AB, BC, CD, and DA are incident in it,

β) " " " AB, BD, DC, and CA " " " ",

γ) " " " AC, CB, BD, and DA " " " ",

ii) A, B, C, and D all are in some instantaneous plane or some vagrant plane.

iii) No three of A, B, C, and D line in one straight line.

iv) No two of the segment satisfying the condition (i intersect.

A straight segment is a <u>side</u> of a quadrilateral if its endpoints are vertices of that quadrilateral. Two sides of a quadrilateral are <u>opposite</u> if no points are incident in both of the two sides. Two sides of a quadrilateral are <u>adjacent</u> if they are not opposite. A quadrilateral is a <u>parallelogram</u> if its opposite sides are parallel. A quadrilateral is a <u>rectangle</u> if its adjacent sides are normal.

§12 Congruence

Congruence is an undefined relation which holds between ~~[illegible]~~ straight segments. Congruence will be assumed to obey a number of axioms, ~~[illegible]~~ The first three axioms are:

1) The opposite sides of a parallelogram are congruent.
2) Congruence is symmetrical
3) Congruence is transitive.[34]

<u>Theorem 1</u> The congruence of sequential straight ~~lines~~ segments of a given time system ~~[illegible]~~ can be established.

This is proven from the fact that such sequential straight lines are parallel so that if they are congruent, a parallelogram will exist to establish this congruence.

A point B is the <u>median</u> of ~~[illegible]~~ AC, if B is incident in ~~[illegible]~~ AC and the ~~[illegible]~~ AB and BC are congruent.

<u>Theorem 2</u> The median of any straight segment can be established.

This is ~~[illegible]~~ because any straight segment is a side of some parallelogram, so the transitive property can be used.

The fourth congruence axiom is:

4) If in a triangle ~~[illegible]~~ which is such that all points incident in it are in some instantaneous plane, the median of a given side is incident in the altitude with respect to the given side, then the other two sides are congruent.[35]

Theorem 3 The congruence of instantaneous straight segments can be established.
This is explained in pages 77-79 of Palter.

~~[illegible] of different time systems~~

The main thing that has not been established is congruence between sequential straight segments of different time-systems.

§13 Cartesian Coordinate Systems and Kinematics.

In any time system α a Cartesian coordinate system can be established at any point O. The procedure is as follows. Find three instantaneous straight segments $OO_{\alpha x}, OO_{\alpha y}, OO_{\alpha z}$, which are normal, and which are incident in some moment of the time system α. Also find a sequential straight segment $OO_{\alpha t}$, such that the points incident in it are in a sequential straight line all of whose points are incident in stations of O in the time system α. $OO_{\alpha x}$ may be taken as a segment of unit length and $OO_{\alpha t}$ may be taken as a segment of unit length. Any point can now be given coordinates $(x_\alpha, y_\alpha, z_\alpha, t_\alpha)$ based upon the axes $OO_{\alpha x}, OO_{\alpha y}, OO_{\alpha z}$, and $OO_{\alpha t}$.

Consider any other time system β. Axes $OO_{\beta x}, OO_{\beta y}, OO_{\beta z}, OO_{\beta t}$ can be chosen to give coordinates $(x_\beta, y_\beta, z_\beta, t_\beta)$, and may be chosen such that $y_\alpha = y_\beta$ and $z_\alpha = z_\beta$. $OO_{\alpha x}$ can be maintained as of unit length for the coordinates x_β, y_β, and z_β as well as those of x_α, y_α and z_α. However $OO_{\beta t}$ must be used as a unit length for the coordinate t_β.[36]

It can be shown that the following coordinate transformation must hold:

$$
\begin{aligned}
x_\beta &= \Omega_{\alpha\beta}\, x_\alpha + \Omega'_{\alpha\beta}\, t_\alpha \\
y_\beta &= y_\alpha \\
z_\beta &= z_\alpha \\
t_\beta &= \Omega''_{\alpha\beta}\, t_\alpha + \Omega'''_{\alpha\beta}\, x_\alpha .
\end{aligned}
\qquad (1
$$

and

$$
\begin{aligned}
x_\alpha &= \Omega_{\beta\alpha}\, x_\beta + \Omega'_{\beta\alpha}\, t_\beta \\
y_\alpha &= y_\beta \\
z_\alpha &= z_\beta \\
t_\alpha &= \Omega''_{\beta\alpha}\, t_\beta + \Omega'''_{\beta\alpha}\, x_\beta .
\end{aligned}
\qquad (2
$$

[37]

The following conditions can be deduced from the fact that the transformation must be inverses of each other.

$$\frac{\Omega_{\alpha\beta}}{\Omega''_{\beta\alpha}} = \frac{\Omega''_{\alpha\beta}}{\Omega_{\beta\alpha}} = -\frac{\Omega'_{\alpha\beta}}{\Omega'_{\beta\alpha}} = -\frac{\Omega'''_{\alpha\beta}}{\Omega'''_{\beta\alpha}} \quad (3$$

$$= \frac{1}{\Omega_{\beta\alpha}\Omega''_{\beta\alpha} - \Omega'_{\beta\alpha}\Omega'''_{\beta\alpha}} = \Omega_{\alpha\beta}\Omega''_{\alpha\beta} - \Omega'_{\alpha\beta}\Omega'''_{\alpha\beta}. \quad 38$$

Now consider any time system Π, if l is any sequential straight line defined by duration of Π, it can be shown that ~~[illegible]~~ there exist constant $\dot{x}_\alpha$, $\dot{y}_\alpha$, and $\dot{z}_\alpha$ and x_{α_0} y_{α_0} z_{α_0} such that l is the set of all points (with coord.) $(x_\alpha, y_\alpha, z_\alpha, t_\alpha)$ where

$$\begin{aligned}
x_\alpha &= x_{\alpha_0} + \dot{x}_\alpha t_\alpha \\
y_\alpha &= y_{\alpha_0} + \dot{y}_\alpha t_\alpha \\
z_\alpha &= z_{\alpha_0} + \dot{z}_\alpha t_\alpha \\
t_\alpha &= \text{any real number.}
\end{aligned}$$

And also constant $\dot{x}_\beta$, $\dot{y}_\beta$, $\dot{z}_\beta$, x_{β_0}, y_{β_0}, and z_{β_0}

$$\begin{aligned}
x_\beta &= x_{\beta_0} + \dot{x}_\beta t_\beta \\
y_\beta &= y_{\beta_0} + \dot{y}_\beta t_\beta \\
z_\beta &= z_{\beta_0} + \dot{z}_\beta t_\beta \\
t_\beta &= \text{any real number}
\end{aligned}$$

The following relation can then be deduced from (1.

$$\dot{x}_\beta = \frac{\Omega_{\alpha\beta}\,\dot{x}_\alpha + \Omega'_{\alpha\beta}}{\Omega''_{\alpha\beta} + \Omega'''_{\alpha\beta}\,\dot{x}_\alpha}$$

$$\dot{y}_\beta = \frac{\dot{y}_\alpha}{\Omega''_{\alpha\beta} + \Omega'''_{\alpha\beta}\,\dot{x}_\alpha}$$

$$\dot{z}_\beta = \frac{\dot{z}_\alpha}{\Omega''_{\alpha\beta} + \Omega'''_{\alpha\beta}\,\dot{x}_\alpha} \,. \qquad 39$$

If π is identified with α, then the velocity of α in β is found, that is,

$$V_{\beta\alpha} = \frac{\Omega'_{\alpha\beta}}{\Omega''_{\alpha\beta}}.$$

Also, if π is identified with β, then the velocity of β in α is found, that is

$$V_{\alpha\beta} = -\frac{\Omega'_{\alpha\beta}}{\Omega_{\alpha\beta}} \,. \qquad 40$$

If the positive direction of time axes are chosen so that they go forward in time, then the following are the additional axioms for congruence.

5) If $V_{\alpha\beta} + V_{\beta\alpha} = 0$, then $OO_{\alpha\tau}$ and $OO_{\beta\tau}$ are congruent.

6) If a velocity of magnitude U in α and normally transverse to the direction of β in α is represented by the velocity $(V_{\beta\alpha}, U')$ in β, where U' is normally transverse to the direction of α in β, then a velocity of magnitude U in β and normally transverse to the direction of α in β is represented by the velocity $(V_{\alpha\beta}, U')$ in α, where U' is normally transverse to the direction of β in α.[41]

It can now be proven that there is an absolute constant K such that for any pair of time systems α and β

$$\frac{V_{\alpha\beta}^2}{1-\Omega_{\alpha\beta}^{-2}} = K$$

where the unit lengths for the two systems are congruent.[42]

Suppose $K = \infty$; this is called the parabolic case. The ~~[illegible]~~ coordinate transformation are then

$$x_\beta = x_\alpha - V_{\alpha\beta}\, t_\alpha$$
$$Y_\beta = Y_\rho$$
$$z_\beta = z_\alpha$$
$$t_\beta = t_\alpha$$

These are the Galilean transformations of classical physics.[43]

Suppose $K = c^2$; this is called the hyperbolic case. The coordinate transformation are then

$$x_\beta = \Omega_{\alpha\beta}\,(x_\alpha - V_{\alpha\beta}\, t_\alpha)$$

$$Y_\beta = Y_\alpha$$

$$z_\beta = z_\alpha$$

$$t_\beta = \Omega_{\beta\alpha}\left(t_\alpha + \frac{V_{\beta\alpha}\, x_\alpha}{c^2}\right)$$

where $\Omega_{\alpha\beta} = (1 - V_{\alpha\beta}^{\,2}/c^2)^{-1/2}$.

These are the Lorentz transformations of ~~[illegible]~~ ~~the~~ Einsteins special theory of relativity.[44]

Notes: Part I

1 Whitehead, *Science and the Modern World* (SMW), p 150.
2 SMW pp 153-154.
3 SMW p 151.
4 SMW p 152.
5 Whitehead, *Process and Reality* (PR), p 224
6 PR p 224
7 PR pp 224-25
8 PR p 225
9 PR p 225
10 PR p 215
11 PR p 215
12 PR p 215
13 PR p 216
14 PR p 216
15 Whitehead and Russell, *Principia Mathematica* (PM), *14

Notes Part II

1 PR p 73
2 PR p 375
3 PR p 73
4 PR p 376
5 Cf. Palter, _Whitehead's Philosophy of Science_, p 28.
6 Whitehead, _Adventures of Ideas_ (AI), pp 195-96.
7 PR p 77
8 PR p 77
9 PR p 82
10 PR p 334
11 PR p 336
12 PR p 346
13 PR pp 348-49
14 Cf Palter pp 45-47
15 Palter pp 44-47.
16 PR pp349-53
17 Cf. Palter p 56
18 Cf Palter pp 57-59
19 Cf Palter p 56
20 Cf Palter p 59
21 Cf Palter p 60
22 Cf Palter p 60
23 Palter p 60
24 Cf Palter p 61
25 Cf Palter pp 63, 65.
26 Cf Palter p 62
27 Cf Palter p 69
28 Cf Palter p 69
29 Cf Palter p 70
30 Palter p 71
31 Cf Palter pp 71-72
32 Cf Palter p 73
33
34 Cf Palter p 76

35 Palter, p 77
36 Palter, pp 79-80
37 Palter, p 80
38 Palter, p 81
39 Palter, p 81
40 Palter, p 81
41 Palter, p 82
42 Palter, p 83
43 Palter, p 85
44 Palter, p 83