On intensional functions of functions

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The definition of intensional and extensional functions of functions in Principia Mathematica, by Whitehead and Russell.

On p. 72 of the 2nd edition they define

“A function is called extensional when its truth-value with any argument is the same as with any formally equivalent argument.”

And on p. 73

“A function is called intensional if it is not extensional.”

They had defined on p. 72

“Two propositional functions are said to be formally equivalent when they are equivalent (have the save truth value) with every possible argument.”

An example.

On p. 73.

“’x is a man’ always implies ‘’x is mortal’” is an extensional function of ‘x is a man’

because we may substitute ‘x is a featherless biped’ (or any other function with the equivalent extension) for ‘x is a man’

But it is not necessary that in

“Tom believes ’x is a man’ always implies ‘’x is mortal’”, that we can substitute ‘x is a featherless biped’ (or any other function with the equivalent extension) for ‘x is a man’.

For Tom may not know all featherless bipeds are men.

The problem.

It seems to me the intensional functions of functions (at least in every example that I am aware of) arise from incomplete or erroneous information. If we had a complete and completely accurate inventory of facts (and we could access these facts instantly as well) , then there would be no intensional functions of functions. They are a consequence of our incomplete and fallible knowledge – but, as such, unavoidable.

Do classes solve the problem?

Trying to go to sleep last night, I thought. "'x ∈ {x | x is a man}' implies 'x ∈ {x | x is a mortal}'" is an intensional function of 'x ∈ { x | x is a man}' . Because Tom may not believe 'x ∈ {x | x is a man}' is equivalent to 'x ∈ {x | x is a featherless biped}' either. What is intensional is 'believes'! We can, though form the classes { x } Tom believes x is a man} and {x | Tom believes x is a featherless biped}.
But, chances are these are equal for Tom, i.e. Tom doesn't have in mind any particular exception to the prior assertion. But Tom would assert '(∃x)((x ∈ {x | x is a man} & (x ~∈ { x | x is a featherless biped})) v (∃x)((x ~∈ {x | x is a man} & (x ∈ { x | x is a featherless biped})).

Trying to apply *20.01

Now letting f(alpha) be Tom believes (alpha ~= {x | x is a featherless biped}) (where alpha = { x | x is a man})

It seems (though, I admit, a little confusing). That f here, even though defined as in *20.01 on the class of men, is still intensional!

Does x ∈ { x | fx } = fx?

Yes, but this is not sufficient. Whitehead and Russell wanted to be able to talk about functions of classes in addition to class membership!

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