Toward An Analysis of Quantification

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Assumptions:

- I'm first examining only the propositional attitude of belief – I will examine the others later.
- I will not assume people think logically.
- I will assume (just for simplicity's sake) that objects of propositional attitudes can be expressed F(a, b, …). or (x)F(x, b), etc.
- These propositional attitudes will be analyzed into 1st order predicate logic.
- The propositional attitudes themselves need not obey the rules of logic, but their analysis do.
- I am here considering only universal quantification.
- Existential quantification can be defined in terms of universal – except in an empty universe.
- The variables occurring in objects of expressed propositional attitudes are just words like other words.

Examination of a simple case using real numbers.

S believes (x)(y)(x + y = y + x)

- Let's assume we understand '+' and '=' for now & just focus on quantifiers and variables.
- One might think it asserts that if one substitutes any name or description of a real number for x and y, then x + y = y + x becomes true.
- But that is insufficient. There are some real numbers that cannot be named or described.
- That is good (for our example) as that occurs in other uses of variables also.
- We learn to use variables from cases where there are names, but generalize beyond that.

It looks like it needs to be analyzed as:

- There are symbols X, Y, PLUS, and EQUALS, such that the relation belief_r holds between X, Y, PLUS, EQUALS, and EQUALS(PLUS(X, Y), PLUS(Y, X))
- X has a variable_symbol_r relation to the set of real numbers.
- Y has a variable_symbol_r relation to the set of real numbers.
- PLUS has a symbol_relation to the binary function +
- EQUALS has a symbol_relation to the binary relation =

S would assert **EQUALS(PLUS(X, Y), PLUS(Y, X))**

I have indicated variables by yellow highlighting. (I'm not very good with LibreOffice, or I would have made the variables themselves a different color. This eliminates the need for the leading quantifiers.

Then if S asserts **EQUALS(PLUS(X, Y), PLUS(Y, X))**

We have

- There are symbols X, Y, PLUS, and EQUALS, such that the relation belief_r holds between X, Y, PLUS, EQUALS, and EQUALS(PLUS(X, Y), PLUS(Y, X))

- X has a variable_symbol_r relation to the set of real numbers.
- Y has a variable_symbol_r relation to the set of real numbers.
- PLUS has a symbol_relation to the binary function +
- EQUALS has a symbol_relation to the binary relation =
- **EQUALS** expresses EQUALS for S
- **PLUS** expresses PLUS for S
- **X** expresses X for S
- **Y** expresses Y for S

A different color could be used for existential quantification, and all statements can be transformed into prenex form – so this should be sufficient though often not convenient. The "prenex" order could be indicated by the lexicographical order of the variables.

For example S expresses **EQUALS**(X , **4**)

becomes

- There are symbols X, 4, and EQUALS, such that the relation belief_r holds between X, 4, and EQUALS(X, 4)
- X has a existential_variable_symbol_r relation to any member of the set of real numbers.
- FOUR has symbol relation to 4
- EQUALS has a symbol_relation to the binary relation =
- **EQUALS** expresses EQUALS for S
- **4** expresses FOUR for S
- **X** expresses X for S