# ABSTRACTS

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## SOLUTION TO A PROBLEM ABOUT PROPOSITIONAL FUNCTIONS: *PRINCIPIA MATHEMATICA* WITH MODALITY

The logic of the first edition of *Principia Mathematica*, taken as an intensional logic, is not completely definite. However, it is by no means unreasonable to suppose that suitable additions might produce an acceptable general intensional logic, the only real competitor in the field being the (unfinished) Church–Frege logic of sense and denotation. The ramified theory of types is indeed complicated, but it is the only developed general logic that deals with all the logical paradoxes, extensional and intensional, as well with Russell's *Paradox about Propositions* (a.k.a. "The Russell–Myhill Antinomy").

If we take propositional functions in the lowest type to be functions from individuals to propositions, a problem arises. Anderson [RIL] shows that this idea leads to a paradox. Even a necessarily asymmetric propositional function will be identical with its converse if there is only a single individual in the domain. Hence the belief that such a function applies to what it applies to will be identical to the belief that the function is asymmetric. Anderson's proposed repair takes propositional functions to be pairs consisting of equivalence classes of formulas and the functions they determine. The proposal is artificial and is in any case unacceptable since the formal system as presented is inconsistent (as shown by Enrico Martino). We show that a simpler and apparently satisfactory solution is available if modality is added to the logic of *PM*. Russell would not consent to this addition, but our current interest is less historical than logico-philosophical. The underlying intensional logic is based on a modification of Church's work on the intensional logic implicit in *Principles of Mathematics* [RSTT, RTIP]. The resulting logic is somewhat simpler than Church's and avoids the need for an additional connective of "four-line equality", with its associated circumlocutions.

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Irving H. Anellis
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## DID THE *PRINCIPIA MATHEMATICA* PRECIPITATE A "FREGEAN REVOLUTION"?

I begin by asking whether there was a Fregean revolution in logic, and, if so, in what did it consist. I then ask whether, and if so, to what extent, Russell played a decisive role in carrying through the Fregean revolution, and, if so, how. A subsidiary question is whether it was primarily the influence of the *Principles of Mathematics* or the *Principia Mathematica*, or perhaps both, that stimulated and helped consummate the Fregean revolution. Finally, I examine cases in which logicians sought to integrate traditional logic into the Fregean paradigm, focusing on the case of Henry Bradford Smith. My proposed conclusion is that there were different means adopted for rewriting the syllogism, either in terms of the logic of relations, or in terms of the propositional calculus, or as formulas of the monadic predicate calculus. This suggests that the changes implemented as a result of the adoption of the Russello-Fregean conception of logic could more accurately be called by Grattan-Guinness's term convolution, rather than revolution.

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Patricia Blanchette
Notre Dame University
FROM LOGICISM TO METATHEORY

Despite the importance of *Principia Mathematica* for the development in Göttingen of the fundamental concepts of modern metatheory, Russell himself never formulates anything very like a modern completeness question, and never engages in the kinds of metatheoretic work that seem to us now to apply straightforwardly to his system. He even seems confused about the application of independence-proof techniques to systems of logic. One explanation of this distance between Russell and his successors is given by Dreben, van Heijenoort, Goldfarb, and Ricketts, who hold that Russell, together with Frege, was barred from making sense of metatheoretical questions by his universalist conception of logic. I'll suggest here that this cannot be right. Neither Frege nor Russell, I'll argue, occupies a position from which metatheory is problematic. They do, on the other hand, share a conception of logic from which completeness as we now understand it would not have been a natural criterion by means of which to assess a system of logic.

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Charles S. Chihara
University of California, Berkeley
THE BURGESS-ROSEN REJECTION OF ALL NOMINALISTIC RECONSTRUCTIONS: *PRINCIPIA MATHEMATICA* AS A TEST CASE

This paper is concerned with the Burgess-Rosen argument that is purported to show that all nominalistic reconstructions of mathematics accomplish almost nothing of value. That argument is supposed to show, in particular, that all nominalistic reconstructions do not succeed in making any advancement in science or mathematics and seem only good for imagining what an alien mathematics might be like. This paper makes the case that, by the Burgess-Rosen line of reasoning, *Principia Mathematica* should also fail to make an advancement in science or mathematics. It is shown in this paper that such a conclusion is contradicted by the judgment of eminent scholars and logicians.

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Ryan Christensen
Brigham Young University
PROPOSITIONAL QUANTIFICATION

Ramsey defined truth in the following way: $x$ is true iff $\exists p\,(x = [p]\ \&\ p)$. This definition is ill-formed in standard first-order logic, so it is normally interpreted using substitutional or some kind of higher-order quantifier. I argue that these quantifiers fail to provide an adequate reading of the definition, but that, given certain adjustments, standard objectual quantification does provide an adequate reading.

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Jolen Galaugher
McMaster University
WHY THERE IS NO FREGE-RUSSELL DEFINITION OF NUMBER

Whether or not Russell's conception of the cardinals should be viewed as akin to Frege's is a matter of both historical and topical importance. It is of historical importance insofar as points of divergence between Frege's and Russell's definitions of the cardinals illuminate more fundamental differences in their logicist projects on the very point on which they are supposed to agree, namely, the logicization of arithmetic. It is of contemporary topical importance, since the problem persists that such an extensional definition of number fails to produce any satisfactory notion of numbers as "set theoretic objects". It has been acknowledged that while Frege simply accepted that the classes by which numbers are defined are logical objects, Russell was concerned with the metaphysical status of abstracta resulting from definition by abstraction. This apparently philosophical consideration did partially motivate Russell's departure from Frege's "… intensional theory of classes [on which] he regards the number as a property of the class-concept, not of the class in extension" (*PM*, S494). However, it still remains to give a positive account of the definition of number that can be properly attributed to Russell, but not to Frege — the one which takes stock of the unique features of the logic underlying Russell's own logicist programme. My aim will be to consider how further inspection of Russell's views in the *PM* exhibits the logical import of the definition of classes by propositional functions.

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Sébastien Gandon
Université Blaise Pascal
*PRINCIPIA MATHEMATICA*, PART VI: RUSSELL AND WHITEHEAD ON QUANTITY

The article aims at providing an introduction to Russell and Whitehead's neglected mature theory of magnitude, presented in the last published part of *Principia Mathematica*. I intend to show that *Principia*, VI, is the apex of a line of thought whose beginning goes back to the time of Russell's first works on the theory of relations, in 1900. But I insist as well on Whitehead's own important contribution. At the end, I address a more general problem: how to articulate this quantitative doctrine of numbers with Russell's and Whitehead's logicist stance?

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Warren Goldfarb
Harvard University
RUSSELL'S LOGIC AND WITTGENSTEIN

TBA

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Ivor Grattan-Guinness
Middlesex University
LOGICISM IN THE CONTEXT OF THE DEVELOPMENT OF SET THEORY

Set theory began to be developed on a large scale from the late 1890s onwards; for example, it was part of the mathematical logic that grounded logicism, and for convenience much of *Principia Mathematica* was elaborated in its terms. Several different parts and features of set theory became prominent, and logicism was supposed to embrace all of them. In practice, though, how many did it tackle? (1) Point set topology; (2) Applications to real-variable

mathematical analysis (measure theory, functional analysis and integral equations); (3) Applications to complex-variable mathematical analysis; (4) The axioms of choice and their consequences; (5) Transfinite arithmetic; (6) General set theory and the absolute; (7) Order-types; (8) Topology, especially dimension theory; (9) Axiomatisation of set theory; (10) Relationship of set theory to logic; (11) Relationship of set theory to model theory.

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Nicholas Griffin
McMaster University
WHATEVER HAPPENED TO GROUP THEORY?

It is a little known fact that Russell, in his first treatment of mathematics after he discovered the work of Peano, gave a prominent place to group theory. In many ways, given the work he had been doing prior to discovering Peano, this was an entirely natural thing for him to do. The surprise is not that he offered an account of group theory but that he abandoned it so quickly without comment. In this paper I consider two questions: (1) Why did he abandon it? (2) How might his work have been different if he had not done so?

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Leila Haaparanta
University of Tampere
ON THEORIES OF JUDGMENT 1910–20

The present paper discusses views on judging and judgment put forward 1910–20. The focus will be on Bertrand Russell's, Edmund Husserl's and Gottlob Frege's views. The texts that will be discussed in more detail are Russell's *Theory of Knowledge* (1913), Husserl's *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie I* (1913) and Frege's article titled "Der Gedanke" (1918). Some comparisons will be made between the three philosophers that show surprising similarities between Russell's and Husserl's views. The paper will also pay special attention to the views on truth that Russell, Husserl and Frege defend in their writings in the 1910s.

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Brice Halimi
Paris Ouest University
AMBIGUITY OF TYPES

My aim is to examine logical types in *Principia Mathematica* from two (partly independent) perspectives. The first one pertains to the ambiguity of the notion of logical type as introduced in the Introduction (to the first edition). I claim that a distinction has to be made between types as called for in the context of paradoxes, and types as logical prototypes. The second perspective bears on typical ambiguity as described in the Prefatory Statement, inasmuch as it lends itself to a comparison with specific systems of modern typed λ-calculus. In particular, a recent paper shows that the theory of logical types can be formalized in the way of a λ-calculus. This opens the avenue of an interesting reconciliation between type theories in the Russellian sense of the word, and type theories in the modern sense. But typical ambiguity is left aside in the paper. I would like to extend the suggestion by taking up the question of typical ambiguity, still in the realm of typed λ-calculus.

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A.P. Hazen
University of Alberta
THE VARIABLE IN METAPHYSICS, SEMANTICS, RUSSELL, AND HIS SUCCESSORS

Variables in formulas are unproblematic: typically italic letters from the end of the alphabet. When Russell used the word “variable”, however, he usually meant not the letter but the corresponding constituent of a non-linguistic proposition, the entity or quasi-entity the letter stands for. This he found philosophically perplexing. Both the “substitutional” theory he formulated immediately after writing “On Denoting” and some of the new material in the second edition of *Principia Mathematica* can be seen as efforts to do without variables in this sense, and the same project of “ontological reduction” can be seen as motivating details in Quine’s *Mathematical Logic* of 1940.

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Arik Hinkis
Independent Scholar, Israel
THE CANTOR-BERNSTEIN THEOREM IN *PRINCIPIA MATHEMATICA*

We survey three proofs of the Cantor-Bernstein theorem (CBT) included in *Principia Mathematica* (*PM*). The first was adapted from Zermelo (1906-08), the second from Borel (1898) and the third recasts the first in the language of cardinal numbers. We discuss the impact of the elimination of the axiom of reducibility, in the second edition of *PM*, upon the first proof. This paper should be experienced as an excursion into the landscape of *Principia Mathematica*.

Harold Hodes
Cornell University
REMARKS ON RAMIFIED-TYPE LANGUAGES AND LOGIC IN THE STYLE OF *PRINCIPIA MATHEMATICA*

This paper starts with formalizations of simple-type languages, and of ramified-type languages (in the spirit of Whitehad and Russell). It presents two arguments for preferring ramification to simplicity. The first originates from conception of propositions and propositional functions as individuated in a way closely reflected by the syntactic structure of formulas. Such a conception, applied to simple-type languages, seems to lead to several paradoxes, variations on Russell's propositional paradox. These paradoxes don't arise for ramified-type languages; one may take that to count in favour of ramification. I will briefly consider a second, more speculative, reason for ramification, one that originates in the Whitehead/Russell metaphysics: the need to analyze away apparent commitments to propositions and propositional functions. Doing so would seem to require a substitutional interpretation of higher-order quantification, which in turn requires ramification. Then I will consider a version of Grelling's paradox, due to Copi, that arises within our ramified-type logic supplemented by the axioms of reducibility when we allow some modest semantic machinery into a ramified-type language. Contra Copi, this doesn't show that the axioms of reducibility bring on inconsistency. But it does undercut the second reason for ramification, and show that the "no class" interpretation of ramified-type logic is required.

Reinhard Kahle
Universidade Nova de Lisboa
THE *PRINCIPIA* IN GÖTTINGEN

The first edition of the *Principia Mathematica* was read in Göttingen in David Hilbert's circle starting from 1914. We will present some material showing the discussion of the *Principia* in Göttingen. Most of this material is already published, in part by Peckhaus [Pec90] and Mancosu [Man03]. The aim of this talk is to advertise these important documents in the history of modern mathematical logic and to put them into the context of Hilbert's foundational research.

David Kaplan
University of California, Los Angeles

TBA

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Kevin Klement
University of Massachusetts

*PM*'S CIRCUMFLEX, SYNTAX AND PHILOSOPHY OF TYPES

Along with offering an historically-oriented interpretive reconstruction of the syntax of *PM* (first edition), I argue for a certain understanding of its use of propositional function abstracts formed by placing a circumflex on a variable. I argue that this notation is used in *PM* only when definitions are stated schematically in the metalanguage, and in argument-position when higher-type variables are involved. My aim throughout is to explain how the usage of function abstracts as "terms" (loosely speaking) is not inconsistent with a philosophy of types that does not think of propositional functions as mind- and language-independent objects, and adopts a nominalist/substitutional semantics instead. I contrast *PM*'s approach here both to function abstraction found in the typed λ-calculus, and also to Frege's notation for functions of various levels that forgoes abstracts altogether, between which it is a kind of intermediary.

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Gregory Landini
University of Iowa

*PRINCIPIA MATHEMATICA*: 100 YEARS OF (MIS)INTERPRETATION

*Principia Mathematica* has been subject to many rival interpretations in the one hundred years since the publication of the first of its three volumes in 1910. The fourth volume never appeared. Russell's own reflections and the new Introduction and appendices of his 1925 second edition fanned an already broad firestorm of controversy. In the light of Russell's voluminous *Nachlass* preserved by the Bertrand Russell Research Centre and Archives, this paper shows how we may finally set the record straight.

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James Levine
Trinity College, Dublin
RUSSELL ON TYPES AND UNIVERSALS

In recent years, Gregory Landini has defended an increasingly influential interpretation of *Principia Mathematica* (*PM*) according to which Russell and Whitehead recognize no typed ontology at all, and instead hold that all entities — including all particulars and universals — are "individuals". In particular, there are, according to Landini two styles of variable in *PM* : "individual variables", which are understood "objectually", and which take absolutely all entities, including all particulars and universals, as their values; and "predicate variables", which are understood "substitutionally" (in the contemporary sense), as "dummy schematic letters", which are to be replaced by open sentences. I present a number of considerations against this sort of interpretation. First, Landini and those following him, including Kevin Klement and Graham Stevens, cite passages from Russell's writings at the time of *PM* in which he holds (as in his earlier *Principles of Mathematics*) that predicates and relations can function either "as subjects" or "as adjectives" or "verbs" (respectively) in order to support the view that Russell also holds that "individual variables" are absolutely unrestricted. I argue, however, that one may regard universals as capable of such a "two-fold" use without also holding that any variable occurring in a subject-position in a propositional function is absolutely unrestricted; that in his 1913 manuscript *Theory of Knowledge*, Russell clearly accepts such a position; and also that there are some indications — albeit not as clear as the 1913 manuscript — that this is also Russell's view in his writings immediately following *PM* in 1910-11. Moreover, I argue that comparing passages from Russell's 1906 manuscript "The Paradox of the Liar" with those writings immediately following the publication of *PM* supports the view that at that time, Russell does not regard predicates and relations as "individuals", but instead regards them as entities of different types than particulars. With regard to the view that in *PM*, Russell and Whitehead accept the "substitutional" interpretation of predicate variables, and simply identify "propositions" with sentences and "propositional functions" with open sentences, I argue that it not only would make it impossible for Russell (who would not countenance infinitary sentences) to sustain his commitment to Cantor's theory of the transfinite, but also attributes to Russell views he could accept only after Wittgenstein had convinced him to reject the "multiple-relation" view of judgment. However, I conclude by arguing that even though Russell does not in *PM* simply identify propositions

with open sentences, he himself recognizes a problem in reconciling his "multiple-relation" view of judgment with Cantor's theory of the transfinite.

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Bernard Linsky
University of Alberta
THE SECOND EDITION OF *PRINCIPIA MATHEMATICA*

The Bertrand Russell Archives hold all of the manuscripts and a large body of notes for the material that Russell added for the 1925–27 second edition of *Principia Mathematica*. Using this material, and the correspondence from this period, it is possible to reconstruct Russell's preparation of the "second edition". Russell, after consulting with Whitehead, began with a long manuscript, "The Hierarchy of Propositions and Functions", which Russell had intended to simply insert into the original Introduction. That manuscript was revised and divided into the eventual "Introduction to the Second Edition" and three Appendices; A, B and C. A significant issue of technical logic concerning Appendix B can be resolved by study of the manuscript of the page where an error occurs, and broader interpretive questions about the second edition can be illuminated by consideration of the draft and unused notes.

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Ed Mares
Victoria University of Wellington
REAL VARIABLES

In *Principles of Mathematics*, Russell holds that propositional variables (or open formulae as we would now say), when asserted, express classes of propositions. This view is retained in the first edition of *Principia* — in the wake of the rejection of classes and propositions — in which asserted open formulae are said to be ambiguous in very much the same sense. But in a manuscript in 1912 Russell claims that open formulae express logical forms, and in the introduction to the second edition of *PM* he reinterprets all of the open formulae asserted in *PM* as standing for closed formulae. This paper investigates why Russell abandoned his original view, a view which seems from a narrowly logical point of view quite coherent regardless of which of the main interpretations of propositional functions and the other elements (or non-elements) of Russell's ontology.

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Conor Mayo-Wilson
Carnegie Mellon University
RUSSELL ON LOGICISM AND COHERENCE

Until recently, many philosophers had suggested that Russell's logicist project, if successful, would be important primarily because it implies that mathematical theorems possess desirable epistemic properties often attributed to logical theorems, such as a prioricity, necessity, and certainty. However, Andrew Irvine and Martin Godwyn argue that such philosophers have failed to understand Russell's primary motivations for logicism, namely, that in reducing mathematics to logic one (a) facilitates mathematical discovery through the creation of new concepts and systematization of existing ones, and (b) improves mathematical explanations. This paper takes Irvine and Godwyn's work as a starting point, and argues that Russell never intended to argue that logicism implies mathematical theorems inherit the traits of a prioricity, necessity, and/or certainty from logical ones. Contra Irvine and Godwyn, however, I argue that Russell thought a systematic reduction of mathematics increases the certainty of known mathematical theorems (even basic arithmetic) facts by showing mathematical knowledge to be coherently organized. The paper outlines Russell's theory of coherence, and discusses its relevance to logicism and the certainty attributed to mathematics.

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Roman Murawski
Adam Mickiewicz University
ON CHWISTEK'S PHILOSOPHY OF MATHEMATICS

Leon Chwistek (1894–1944) is known mainly for his logical works, in particular for his simplification of Whitehead and Russell's theory of types. His logical investigations however were — as it was the case by some Polish logicians — connected with his philosophical ideas concerning logic and mathematics. Moreover, they were in a sense motivated by those ideas. Building semantics, he wanted to overcome philosophical idealism and was against the conception of an absolute truth. He did not content himself with solving particular definite fragmentary problems but — similarly to Lesniewski — attempted to construct a system containing the whole of mathematics. The paper is devoted to the presentation of Chwistek's philosophical ideas concerning logic and mathematics. According to him human knowledge is neither full nor absolute. He accepted the principle of rationalism of knowledge and was against

irrationalism. A way from the difficulties caused by irrationalism and simultaneously a weapon in a struggle against it is formal logic, in particular rational metamathematics founded by him. His epistemological views were close to the neopositivism. According to Chwistek an object of a cognition can be only that what is given in an experience. There are however various types of experience. In this way we come to the best known original philosophical conception of Chwistek, namely to his theory of the plurality of realities. The main feature of his philosophy was nominalism which found full expression in his philosophy of mathematics. He claimed that the object of deductive sciences, hence in particular of mathematics, are expressions being constructed in them according to accepted rules of construction. Geometry is for Chwistek an experimental discipline. He clearly and categorically rejected conventionalism in geometry. Similarly as geometry one should treat also arithmetic, mathematical analysis and other mathematical theories, obtaining in this way a nominalistic interpretation of them. The fate of the philosophical conceptions of Chwistek was similar to the fate of his logical conceptions. The system of rational metamathematics has not been developed by him in detail. He worked on his own conceptions and ideas without any collaboration with other logicians, mathematicians or philosophers. His investigations were not in the mainstream of the development of logic and philosophy of mathematics.

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Raymond Perkins, Jr.
Plymouth State University

INCOMPLETE SYMBOLS IN *PRINCIPIA MATHEMATICA* AND RUSSELL’S “DEFINITE PROOF”

Early in *Principia Mathematica* Russell presents an informal argument that definite descriptions are incomplete symbols — that they function differently than proper names, and that they have meaning only in sentential context. A few pages later he refers to this argument as a “definite proof”. This “proof” is significant not only because it is central to understanding incomplete symbols so vital to Russell’s logicism, but also because it has been taken as evidence of Russell’s supposed carelessness and confusion in philosophy of logic and language. This paper will (1) defend Russell’s “proof” against several charges and thereby rectify some long-standing criticisms of Russell’s semantics, (2) bring out some of Russell’s epistemic and metaphysical ideas connected with his doctrine of names and incomplete symbols and (3) offer some observations on how his *Principia* definitions of descriptions and class-symbols fit into his more general conception of philosophical analysis during his atomistic period.

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Ori Simchen
University of British Columbia
POLYADIC QUANTIFICATION VIA DENOTING CONCEPTS

The question of the origin of polyadic expressivity is explored and the results are brought to bear on Bertrand Russell's 1903 theory of denoting concepts, which is the main object of criticism in Russell's "On Denoting". It is shown that, appearances to the contrary notwithstanding, the background ontology of the earlier theory of denoting enables the full-blown expressive power of first-order polyadic quantification theory without any syntactic accommodation of scopal differences among denoting phrases such as "all φ", "every φ", and "any φ" on the one hand, and "some φ" and "a φ" on the other. The case provides an especially vivid illustration of the general point that structural (or ideological) austerity can be paid for in the coin of ontological extravagance.

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Peter Simons
Trinity College, Dublin
$+_c$ , OR WHY WHITEHEAD AND RUSSELL DIDN'T NEED THEIR FINGERS TO ADD UP

Proving wrong Kant's claim that we need intuition in order to do simple addition was a key task for logicism. Showing elementary arithmetical propositions are truths of logic should have been child's play, but Whitehead and Russell take a very long way round, defining addition of cardinal numbers as late as *112. Like multiplication and exponentiation, they require it to be defined for arbitrary classes, even if these are of different types, or overlap. The motivation for this liberality is maximum generality. Tracing the definitions back to their basics enables us to see the ingenious way they do it, but the tortuous route renders addition so opaque that the use of fingers appears much closer to the logic of addition than the *Principia* definition, with implications for the status of Whitehead and Russell's logicism.

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Graham Stevens
Manchester University
LOGICAL FORM IN *PRINCIPIA MATHEMATICA* AND ENGLISH

TBA

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Dustin Tucker
University of Michigan
OUTLINE OF A THEORY OF QUANTIFICATION

Historically, Russell's ramified theory of types has been somewhat neglected, but with good reason. As a foundation of mathematics, it required the infamous axiom of reducibility. As a solution to paradoxes, it seemed unnecessary in light of Ramsey's distinction between semantical and set-theoretical paradoxes. As a general theory, it was motivated by the vicious-circle principle, which Gödel convincingly attacked. But several authors, including Alonzo Church, David Kaplan, and Rich Thomason, have suggested ramification as a resolution of a family of paradoxes that fall through the cracks of Ramsey's division. These paradoxes have been largely neglected as well, and what little existing work there is often leaves much to be desired. But so does ramification. Like Tarski's hierarchy of languages, it is much too heavy-handed to do justice to the paradoxes. I thus provide a refinement of ramification's restrictions on propositional quantification, analogous to Kripke's refinement of Tarski's truth predicates. Along the way, I develop a more flexible logic of propositions and an alternate resolution of the paradoxes, using truth-value gaps.

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Alasdair Urquhart
University of Toronto
*PRINCIPIA MATHEMATICA* AND ITS INFLUENCE

This paper investigates the influence that *Principia Mathematica* exerted on the course of logical studies in the century since its publication. It will demonstrate the waning influence of the treatise in the course of the twentieth century, as well as discussing its impact on major logicians such as Tarski, Herbrand and Gödel.

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Russell Wahl
Idaho State University
THE AXIOM OF REDUCIBILITY

The axiom of reducibility plays an important role in the logic of *Principia Mathematica*, but has generally been condemned as an ad hoc non-logical

axiom which was added simply because the ramified type theory without it would not yield all the required theorems. In this paper I examine the status of the axiom of reducibility. Whether the axiom can plausibly be included as a logical axiom will depend in no small part on the understanding of propositional functions. If we understand propositional functions as constructions of the mind, it is clear that the axiom is clearly not a logical axiom and in fact makes an implausible claim. I look at two other ways of understanding propositional functions, a nominalist interpretation along the lines of Landini (1998) and a realist interpretation along the lines of Linsky (1999) and Mares (2007). I argue that while on either of these interpretations it is not easy to see the axiom as a non-logical claim about the world, there are also appear to be difficulties in accepting it as a purely logical axiom.

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Jan Woleński
Jagiellonian University
*PRINCIPIA MATHEMATICA* IN POLAND

Russell says in his *My Philosophical Development*, p. 86: "I used to know of only six people who had read the later parts of the book. Three of these were Poles, subsequently (I believe) liquidated by Hitler. The three other were Texans, subsequently successfully assimilated."

As far as I know nobody has identified the three readers of "later parts" of *PM* from Texas. As far as the matter concerns Poles, we have at least four candidates, namely Jan Śleszyński (1854–1931), Leon Chwistek (1884–1944), Stanisław Leśniewski (1886–1939) and Alfred Tarski (1901–1983). The dates show that none of them was liquidated by Hitler (Leśniewski died in May 1939, Chwistek died in Soviet Union). On the other hand, the Nazis killed several other Polish logicians who could read *PM*, even later parts of this great work, for example Adolf Lindenbaum (1904–about 1941) or Mordechaj Wajsberg (1902–about 1943). *PM* was recommended as an advanced monograph for students of mathematics specializing in mathematical logic and the foundations of mathematics (see guides by Warsaw University). In fact, Leśniewski wanted to translate *PM* into Polish. These facts document how the treatise of Whitehead and Russell was popular in Poland.

As far as the matter concerns Polish investigations related to *Principia Mathematica*, the following lines are relevant: (1) work on the Russell paradox

(Chwistek, Leśniewski, Czeżowski, Tarski); (2) simplifications of the theory of types (Chwistek, Wilkosz); (3) criticism of *PM* (Leśniewski); (4) the theory of syntactic categories and levels of language (Leśniewski, Tarski). Looking from another axis, logical and foundational systems (grand logics) of Chwistek and Leśniewski were a continuation (Chwistek) or response (Leśniewski) to *PM*. Moreover, some more concrete constructions, in particular, the semantic definition of truth, arose in the framework of *Principia Mathematica*.

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Byeong-Uk Yi
University of Toronto
CLASS AND NUMBER IN *PRINCIPIA MATHEMATICA*

This paper examines the no-class theory of *Principia Mathematica*. It clarifies an anomaly in the logic of classes that results from the theory, and gives a reformulation of the theory that removes the anomaly and yields a complete parallelism between the logic of classes and the logic of individuals and propositional functions. The reformulation results from giving a refined account of the notion of predicative function using plural languages. The account yields as theorems of plural logic the strengthened version of the Axiom of Reducibility that implies that formally equivalent predicative functions are identical. The reformulation of the no-class theory suggests that natural numbers and classes can be identified as predicative functions of a special kind.

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Richard Zach
University of Calgary
*PRINCIPIA MATHEMATICA* AND THE DEVELOPMENT OF LOGIC

Even if by the 1920s and 1930s foundational research in mathematics itself was dominated by work in set theory, Whitehead and Russell's work continued to have a profound influence on the development of logic and its applications in philosophy. In mathematical logic, this influence can be traced in particular through the reception of *PM* in David Hilbert's school, which produced not only fundamental work in logic itself, where the influence of *PM* was decisive, but also two textbooks, Hilbert and Ackermann's *Grundzüge der theoretischen Logik* (1928) and Hilbert and Bernays' *Grundlagen der Mathematik* (1934, 1939), which remained standard texts for many years. In philosophy, *PM* was influential in particular through the work of Carnap, who wrote an introduction to

Whitehead and Russell's "new logic" (*Abriss der Logistik*, 1929) and made crucial use of the techniques developed in *PM* in his *Aufbau* (1928) and related works. These developments were not unrelated, and also not isolated.